Material Adjustment Transforms – Practical applications of the Wpt material colour equivalency representation

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Introduction

Advanced iccMAX profiles provide the ability to perform colour management for arbitrary observers and illuminants resulting in sensor excitations. As such colorimetric conversions require adjustment transforms that incorporate observer and illuminant differences. This paper focuses largely on a mathematical approach to the computation, manipulation, and metrics related to sensor excitations representing the generalized concept of the amount of light captured by either the visual system or a digital camera by the means of colour equivalency representation which minimizes colour changes due to differences in observing condition (observer and/or illuminant). This paper first introduces the mathematical formation of sensor excitations and using sensor adjustment transforms in relationship to colour equivalency representations. Then, material adjustment transforms using Waypoint (Wpt) normalization and perceptually uniform material colour equivalency space (WLab) are presented. After that, a few useful metrics based on WLab are introduced to evaluate objects, illuminants, and observers. Finally, observer conversion as well as spectral estimation and manipulation using Wpt are presented.

Sensor Excitations

A collection of sensor excitations is represented as a mathematical vector that quantifies the colour of light for a single picture element (pixel) or point in a scene as seen by an observer. The transformative system of an arbitrary situation going from the k^{th} illuminant reflected off the j^{th} object captured by the i^{th} observer or device resulting in a vector of sensor excitations can be represented by the following linear equation:

$$\boldsymbol{s}_{i,j,k} = \boldsymbol{C}_i \boldsymbol{O}_j \boldsymbol{I}_k \tag{1}$$

where

$\boldsymbol{s}_{i,j,k}$ is a vector of sensor excitations for the jth object for the observing condition defined by the ith observer under the kth illuminant

$\boldsymbol{C}_i$ is a matrix with rows representing normalized cone fundamentals, color matching functions, or sensor sensitivities at discrete wavelengths for the ith observer

$\boldsymbol{O}_j$ is a Donaldson matrix that describes relationships between incident and reflected light from the jth object with diagonal entries corresponding to spectral reflectance and off-diagonal entries corresponding to fluorescent reflectance

$\boldsymbol{l}_k$ is a vector representing intensities of light at discrete wavelengths for the kth illuminant

Sensor Adjustment Transforms and Colour Equivalency Representations

A linear Sensor Adjustment Transform (SAT) can be expressed as applying a linear matrix to a sensor excitation vector for an object with one observing condition (defined by observer and illuminant) to another observing condition preserving some sense of colour equivalency. This is expressed by the following:

$$\boldsymbol{s}_{2,j,2} \cong \boldsymbol{M}\boldsymbol{s}_{1,j,1} \tag{2}$$

where

$\boldsymbol{M}$ is a matrix that adjusts a sensor excitation for one observing condition to predict some form of colour equivalent sensor excitation result for a second observing condition

Various approaches are used for defining matrix $\boldsymbol{M}$ but considering that some form of colour equivalency is involved it makes sense to decompose this transform into two transforms. One that transforms the sensor excitation into a colour equivalency representation and a second one to transform out of the colour equivalency representation (using and inverse transform). Thus:

$$\boldsymbol{s}_{2,j,2} \cong \left(\boldsymbol{N}_{2,2}\right)^{-1} \boldsymbol{N}_{1,1}\boldsymbol{s}_{1,j,1} \tag{3}$$

where

$\boldsymbol{N}_{i,k}$ is a normalization matrix that transforms a sensor excitation for the ith observer and kth illuminant into a colour equivalency representation vector

White Balancing

White balancing utilizes a colour equivalency representation with a singular definition of white defined as $[1\ 1\ \ldots\ 1]^T$. A white balancing normalization transform involves first

determining white point for an observing condition and defining the normalization matrix as a diagonal matrix transform containing reciprocals of the white point coordinates (von Kries 1970). The white point is determined using an identity matrix for the object in eq. 1 (a perfect reflecting diffuse) to get sensor excitation values for the observer and illuminant.

Chromatic Adaptation Transforms

Chromatic Adaptation Transforms predict corresponding colours found using colour appearance matching experiments (Susstrunk 2005). The colour equivalency space of linear transforms for common colour appearance transforms involve first applying a matrix optimized to predict corresponding colours that sharpens the cone sensitivity functions and then applies white balancing on the sharpened sensor excitations.

Material Adjustment Transforms using Waypoint (Wpt) normalization

The colour equivalency representation for white balancing is based on having a fixed position for the white point with relative scaling used for normalization. The subject of this report is extending the normalization concept to utilize a colour equivalency representation that can be used as a waypoint for defining Sensor Adjustment Transforms that predict how sensor excitations of material objects change due to changes in illuminant and observer. This waypoint equivalency representation has a fixed white point in addition to having an opponency coordinate system that uses normalization to preserve preceptive correlates of lightness, chroma, and hue with Cartesian coordinates referred to as W, p and t.

The Waypoint (Wpt) colour equivalency representation (Derhak & Berns 2015-1) is established by using a perfect reflecting diffusor (PRD represented by Identity $\boldsymbol{O}_j$ matrix) with additional spectral reflectance with attributes of constant perceptive correlates of lightness and chroma for a reference observing condition. Spectral reflectances of physical manifestations of Munsell colours were adjusted to achieve the Munsell Renotation colorimetric aims under Illuminant C for the CIE 1931 standard 2-degree observer. Linear rotational, shearing, and scaling transforms are applied to the sensor excitations of a PRD, reflectances of the set of Munsell colours constant Munsell Value of 5 and Munsell Chroma 6, and reflectances of Munsell colour 3.75Y 8/14 as part of the Wpt normalization process. The reflectances used can be found in Annex A.

The reference Wpt normalization space is defined for the reference observing conditions by using a sequence of linear transforms that accomplish the following steps (with a functional implementation found in Annex section C.1):

1) Sensor excitations for the PRD are rotated around all but the first coordinate axis to align white point with first coordinate resulting in the first coordinate (W) representing grayscale values.
2) The first coordinate is scaled so that the PRD has a W value of 100.
3) The application of the first two steps of the sensor excitations of the reflectances having constant Munsell Value and Chroma are fitted with the equation of a plane and the coeffects of the equation are used to define a transform that shears the plane so that it is orthogonal to the first axis. This results in constant W coordinates corresponding to constant Munsell Value.
4) The application of the first three steps of the sensor excitations of the reflectances having constant Munsell Value and Chroma are fitted with the equation of an ellipse and the coefficients of the equation are used to scale the ellipse so that it becomes a circle. This results in constant distance from the W axis corresponding to constant Munsell Chroma.
5) The application of the first four steps of the sensor excitations of the reflectances for Munsell 3.75Y 8/14 are rotated around the W axis so that it is collinear with the third axis (t).
6) The application of the first five steps of the sensor excitations of the reflectances for Munsell 3.75Y 8/14 are scaled so they have a t value of 100.
7) The second (p) axis is scaled by -1 if the application of the first six steps of the sensor excitations of the reflectances for Munsell 5R 5/6 results in the second axis having a negative value

Wpt normalization of sensor excitations for arbitrary observing conditions (with different observer and/or illuminant) to the Wpt colour equivalency representation is achieved as follows (with a functional implementation found in Annex section C.2):

8) Sensor excitations for the PRD are rotated around all but the first coordinate axis to align white point with first coordinate resulting in the first coordinate (*W*) representing grayscale values.
9) The first coordinate is scaled so that the PRD has a *W* value of 100.
10) The application of the first two steps of the sensor excitations of the reflectances having constant Munsell Value and Chroma are fitted with the equation of a plane and the coeffects of the equation are used to define a transform that shears the plane so that it is orthogonal to the first axis. This results in constant *W* coordinates corresponding to constant Munsell Value.
11) Linear regression is applied to adjusted sensor excitations (of steps 1 through 3) to define a linear transform that minimizes differences of p & t values for the 40 Munsell reflectances thus minimizing differences in chroma and hue.

These four steps are here after referred to as Wpt normalization that uses a function of observer and illuminant to define a matrix that normalizes sensor excitations into the

material colour Wpt colour equivalency representation. An equation which extends eq. 1 to determine Wpt vectors for k^{th} illuminant reflected off the j^{th} object captured by the i^{th} observer is as follows:

$$\begin{bmatrix} W \\ p \\ t \end{bmatrix}_{i,j,k} = \boldsymbol{w}_{i,j,k} = \boldsymbol{A}_{i,k}\boldsymbol{C}_i\boldsymbol{O}_j\boldsymbol{l}_k \tag{4}$$

where

$\boldsymbol{A}_{i,k} = f\left(\boldsymbol{C}_i, \boldsymbol{l}_k\right)$ which is a function of an observer and illuminant that results in a normalization matrix that converts sensor excitations to vectors in the Wpt colour equivalency representation. Instances of this function can be found in Annex C.

Note: As Wpt normalization relies on a normalization of the perceptive corelates of lightness, chroma, and hue it requires that the sensor excitations of the reflectances (found in Annex A) used for normalization clearly express these correlates. This requires that the application of illuminant and observer functions results in sensor excitations of the normalization reflectances that maintain a generally linear relationship to the sensor excitations of the normalization reflectances for the reference observing conditions. These general linear relationships break down when either the illuminant has a spectral power distribution that doesn't cover the entire visible wavelength range (narrow-band) or the observer sensitivity functions are not clearly distinct from one another (effectively having fewer than three distinct sensor functions - which is the case for some colour vision deficiencies).The polar representation (Wch) of Wpt is can be found by applying the function $q()$ as follows:

$$q\left(\boldsymbol{w}_{i,j,k}\right) = \begin{bmatrix} W \\ \sqrt{p^2 + t^2} \\ \arctan\left(p/t\right) \end{bmatrix}_{i,j,k} = \begin{bmatrix} W \\ c_{pt} \\ h_{pt} \end{bmatrix}_{i,j,k} \tag{5}$$

Pairs of Wpt normalization matrices for different observing conditions can be used to define Material Adjustment Transforms (using the pattern shown $\boldsymbol{M}$ in eq. 2) that predict how sensor excitations for an object will change due to differences in illuminant and/or observer. Thus:

$$\boldsymbol{s}_{2,j,2} \cong \left(\boldsymbol{A}_{2,2}\right)^{-1} \boldsymbol{A}_{1,1}\boldsymbol{s}_{1,j,1} \tag{6}$$

Perceptually Uniform Material Color Equivalency Space (WLab)

The Wpt color equivalency space has several desirable properties. It uniquely defines white and grayscale, and it preserves perceptive lightness, chroma and hue. However, equal Euclidean distances between Wpt coordinates do not correspond to equal perceptual colour differences since Wpt normalization represents a linear combination of sensor excitations which have been shown to no be perceptually uniform. Invertible non-linear transforms can be applied to Wpt coordinates resulting in coordinates (waypoint-Lab or WLab coordinates) that have more perceptually uniform attributes between differences in colour coordinates (Derhak & Berns 2015-2). The equations in Annex D provide the means to convert from Wpt coordinates ($\boldsymbol{w}_{i,j,k}$) to perceptually uniform WLab coordinates ($\boldsymbol{c}_{i,j,k}$) represented hereafter as the function $g()$.

$$\boldsymbol{c}_{i,j,k} = g\left(\boldsymbol{w}_{i,j,k}\right) \tag{7}$$

The equations in Annex E provide the means to convert from perceptually uniform WLab coordinates ($\boldsymbol{c}_{i,j,k}$) to Wpt coordinates ($\boldsymbol{w}_{i,j,k}$) represented hereafter as the inverse function $\mathrm{g}^{-1}()$.

$$\boldsymbol{w}_{i,j,k} = g^{-1}\left(\boldsymbol{c}_{i,j,k}\right) \tag{8}$$

An important point to remember is that WLab is a transformation of Wpt coordinates which already have opponency conversion applied. Therefore, WLab perceptual uniformity corrections are applied directly to opponency coordinates resulting in greater numerical stability than other colour spaces (like CIELAB) which perform opponency conversion after non-linear perceptual uniformity corrections are applied to normalized sensor excitations.

WLab Metrics

Because WLab represents a uniform material colour equivalency representation with perceptual uniformity for a reference observing condition it provides a convenient coordinate system for colour comparison as differences due to observer, object, and/or illuminant are represented by Euclidean distances between WLab coordinates. Thus, the following equation represents a colour difference between objects and/or observing conditions:

$$\Delta E_{2,1} = \left| \boldsymbol{c}_{2,2,2} - \boldsymbol{c}_{1,1,1} \right| \tag{9}$$

Various colour metrics are possible by selecting which aspects of the observer, object, and illuminant that vary or remain the same and combining the results as things are varied. The following sub-sections outline a few useful metrics:

Object colour differences

Colour differences between two objects are found when the observer and illuminant are the same (Derhak 2015). Thus:

$$\Delta E_{WLab} = \left| \boldsymbol{c}_{i,2,k} - \boldsymbol{c}_{i,1,k} \right| \tag{10}$$

Object colour inconstancy

An index of object colour inconstancy, Mean Object Inconstancy Index (MOII), is found by taking the mean of the colour differences for the j^{th} object under different observing condition compared to a reference observing condition (Derhak 2015). Thus:

$$MOII = \frac{\sum_i \sum_k \left| \boldsymbol{c}_{i,j,k} - \boldsymbol{c}_{ref,j,ref} \right|}{ik} \tag{11}$$

Object illuminant metamerism

An index of illuminant metamerism (IIM) is found by taking the mean of the colour differences for two objects under different illuminants for the same observer. Thus:

$$IIM = \frac{\sum_k \left| \boldsymbol{c}_{i,2,k} - \boldsymbol{c}_{i,1,ref} \right|}{k} \tag{12}$$

Object observer metamerism

An index of observer metamerism (IOM) is found by taking the mean of the colour differences for two objects observed by different observers compared to the two objects observed by a reference observer under the same illuminant. Thus:

$$IOM = \frac{\sum_i \left| \boldsymbol{c}_{i,2,k} - \boldsymbol{c}_{ref,1,k} \right|}{i} \tag{13}$$

Object metamerism variability

An index of object metamerism variability, Mean Metameric Object Colour Difference (MMOCD), is found by taking the mean of the colour differences for two objects under varying illuminants for multiple observers (Derhak 2015). Thus:

$$MMOCD = \frac{\sum_i \sum_k \left| \boldsymbol{c}_{i,2,k} - \boldsymbol{c}_{i,1,k} \right|}{ik} \tag{14}$$

Illuminant colour rendering

An index of illuminant colour rendering (IR) is found by taking the mean of the colour differences for a set of objects under the k^{th} illuminant compared to the set of objects under a reference illuminant for the same observer. Thus:

$$IR = \frac{\sum_j \left| \boldsymbol{c}_{i,j,k} - \boldsymbol{c}_{i,j,ref} \right|}{j} \tag{15}$$

Illuminant rendering variability

An index of illuminant rendering variability, Illuminant Rendering Equivalency (IRE), is found by taking the mean of the colour differences for varying objects and observers under the k^{th} illuminant as compared to the same set of objects and observers under a reference illuminant (Derhak 2015). Thus:

$$IRE = \frac{\sum_i \sum_j \left| \boldsymbol{c}_{i,j,k} - \boldsymbol{c}_{i,j,ref} \right|}{ij} \tag{16}$$

Observer difference

An index of observer difference, Observer Matching Equivalency (OME), is found by taking the mean of the colour differences for varying objects and illuminants for two observers (Derhak 2015). Thus:

$$OME = \frac{\sum_i \sum_k \left| \boldsymbol{c}_{2,j,k} - \boldsymbol{c}_{1,j,k} \right|}{ik} \tag{17}$$

Note: The colour matching of the two observers is identical when the IOD is zero.

Observer Conversion using Wpt normalization

The use of physiologically based cone fundamentals or camera sensitivity functions in equations or systems based on standardized colour matching functions (like CIELAB or CIECAM) requires a transformation of the cone fundamentals or sensitivity functions that result in colour matching functions that produce “colorimetric values” that have similar properties to that of a standard observer (Derhak 2015).

The application of a Wpt based MAT to cone fundamentals (as shown in the following equation) results in colour matching functions that have several desirable characteristics.

$$\boldsymbol{C}_{std_custom} = \left(\boldsymbol{A}_{std,ref}\right)^{-1} \boldsymbol{A}_{custom,ref} \boldsymbol{C}_{custom} \tag{18}$$

Such desirable characteristics of the resulting colour matching functions include:

- Colour matching of the cone fundamentals or sensitivity functions is preserved since a linear transform is applied.
- The white point is exactly matched for the reference illuminant.
- Perceptive lightness, chroma, and hue are generally preserved since Wpt normalization uses minimization of differences for a well-defined set of spectral reflectances.
- Works well even when the number of custom sensitivity functions is greater than three.

Spectral Estimation and Manipulation using Wpt

The Wpt colour equivalency representation provides a useful tool for spectral estimation and manipulation because it has a linear relationship to spectral reflectance (eq. 4) and it had perceptive correlates of lightness, chroma, and hue. Additionally, linear combinations of spectral reflectance have corresponding linear combinations of Wpt coordinates. One especially useful spectral reflectance decomposition is the Chau decomposition (Chau 1999) that represents spectral reflectance as the sum of scaled non-selective reflectance ($\boldsymbol{I}$) and a characteristic reflectance ($\boldsymbol{R}$) as follows:

$$\boldsymbol{O} = g\boldsymbol{I} + s\boldsymbol{R} \tag{19}$$

As eq. 19 is applied in eq. 4 and the scalar s is varied one finds that the relationship between Wpt coordinates p & t (expressed as h_{pt} or Wpt hue) remains constant. Thus, $\boldsymbol{R}$ in eq. 19 determines Wpt hue, and conversely Wpt hue can be used in specific domains to determine matrix $\boldsymbol{R}$.

Spectral estimation from Wpt is represented by the function $r()$ in eq. 20.

$$\boldsymbol{O}_j = r\left(\boldsymbol{w}_{i,j,k}, \boldsymbol{A}_{i,k}, \boldsymbol{C}_i, \boldsymbol{l}_k\right) \tag{20}$$

The steps to implement estimation function $r()$ are as follows:

1. The Wpt coordinate ($\boldsymbol{w}_{i,j,k}$) is converted to polar notation as Wch using function $q()$ of eq. 5 resulting in coordinate values $W_{i,j,k}$, $c_{i,j,k}$, and $h_{i,j,k}$.
2. A domain specific function of the $h_{i,j,k}$ (Wpt hue) of step 1 is used to determine a characteristic reflectance ($\mathbf{R}_j$) for the hue.
3. Wch values for the characteristic reflectance ($\mathbf{R}_j$) are determined using eq. 4 and eq. 5 resulting in coordinate values W_R, c_R, and h_R.
4. The estimated reflectance for the Wpt starting value of step 1 is then determined using Eq. (19) with the characteristic reflectance ($\mathbf{R}_j$) of step 2 with the scalars g and s being determined directly from a linear relationship with the $W_{i,j,k}$ and $c_{i,j,k}$ values found in step 1 as follows:

$$\begin{bmatrix} g \\ s \end{bmatrix} = \mathbf{M}_\mathbf{R} \begin{bmatrix} W_{i,j,k} \\ c_{i,j,k} \end{bmatrix} \tag{21}$$

where:

$\mathbf{M}_R$ is a matrix associated with the characteristic reflectance $\mathbf{R}_j$ which leverages the linear relationship between Wch values and $\mathbf{R}_j$ from step 3 with the following linear regression:

$$\mathbf{M}_\mathrm{O} = \begin{bmatrix} W_\mathrm{I} & W_\mathrm{R} \\ 0 & c_\mathrm{R} \end{bmatrix}^{-1} \tag{22}$$

where:

W_I is the Wpt W coordinate for the white point (PRD), W_R is the Wpt W coordinate for $\mathbf{R}_j$, and c_R is the Wpt c coordinate for $\mathbf{R}_j$.

An adjustment to a base spectral reflectance is represented by adding the residual difference of the spectral estimation of the base and the spectral estimation of the adjusted Wpt coordinates as follows:

$$\begin{aligned} \boldsymbol{w}_{i,base,k} &= \boldsymbol{A}_{i,k}\boldsymbol{C}_i\boldsymbol{O}_{base}\boldsymbol{l}_k \\ \boldsymbol{O}_{new} &= \boldsymbol{O}_{base} + \left(r\left(\boldsymbol{w}_{i,new,k}, \boldsymbol{A}_{i,k}, \boldsymbol{C}_i, \boldsymbol{l}_k\right) - r\left(\boldsymbol{w}_{i,base,k}, \boldsymbol{A}_{i,k}, \boldsymbol{C}_i, \boldsymbol{l}_k\right)\right) \end{aligned} \tag{23}$$

It is desirable that the spectral estimations have similar characteristics to the base reflectance so that the adjusted reflectances are meaningful. The ability to use domain specific definitions of characteristic reflectances based on Wpt hue is an important aspect of this approach.

Annex A: Wpt Normalization Spectral Reflectance Factors

The following table provides spectral reflectance factors used for Wpt Normalization with rows associated with Munsell Notation chips and columns associated with wavelengths.

Table A.1 — Wpt Normalization Spectral Reflectance Factors

	380nm	385nm	390nm	395nm	400nm	405nm	410nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.12751313	0.12868761	0.14108923	0.15458930	0.16320429	0.16701782	0.16688539
5 R 5/6	0.11796216	0.11961336	0.12925223	0.13971535	0.14565224	0.14831352	0.14824575
7.5 R 5/6	0.11168290	0.11331619	0.12063671	0.12769113	0.13061625	0.13140239	0.13090448
10 R 5/6	0.10098715	0.10186626	0.10471844	0.10716129	0.10781944	0.10793951	0.10753051
2.5 YR 5/6	0.08953364	0.08969376	0.08985388	0.09001399	0.09017411	0.09052383	0.09037476
5 YR 5/6	0.07574633	0.07579202	0.07583772	0.07588341	0.07592911	0.07604643	0.07590959
7.5 YR 5/6	0.06530961	0.06533335	0.06535708	0.06538082	0.06540456	0.06538788	0.06536054
10 YR 5/6	0.05565312	0.05581902	0.05598492	0.05615082	0.05631672	0.05608860	0.05604462
2.5 Y 5/6	0.04650443	0.04683980	0.04717503	0.04751014	0.04784511	0.04805418	0.04791879
5 Y 5/6	0.03957048	0.04007713	0.04058375	0.04109035	0.04159693	0.04237105	0.04213183
7.5 Y 5/6	0.03668714	0.03691875	0.03715037	0.03738198	0.03761360	0.03869815	0.03839452
10 Y 5/6	0.03817243	0.03828879	0.03840515	0.03852151	0.03863788	0.03875424	0.03959883
2.5 GY 5/6	0.04394507	0.04397987	0.04401466	0.04404946	0.04408426	0.04411905	0.04415362
5 GY 5/6	0.04944295	0.04944324	0.04944354	0.04944383	0.04944413	0.04944443	0.04944295
7.5 GY 5/6	0.06316426	0.06316445	0.06316464	0.06316482	0.06316501	0.06316519	0.06316426
10 GY 5/6	0.07859517	0.07865495	0.07871474	0.07877452	0.07891248	0.07898668	0.07902289
2.5 G 5/6	0.07265424	0.07497063	0.07728702	0.07971397	0.08502300	0.08788974	0.08927768
5 G 5/6	0.06965128	0.07451918	0.07938708	0.09225480	0.10162134	0.10743255	0.10957428
7.5 G 5/6	0.07255240	0.07330150	0.07405060	0.08806750	0.09771616	0.10359601	0.10663964
10 G 5/6	0.07137711	0.07668932	0.08200153	0.10002653	0.11456201	0.12296897	0.12742496
2.5 BG 5/6	0.07061577	0.07101322	0.08894306	0.10835921	0.12404616	0.13323403	0.13878737
5 BG 5/6	0.06784482	0.07378653	0.09500809	0.11905868	0.13842605	0.15173143	0.15898610
7.5 BG 5/6	0.07053930	0.07325998	0.09797536	0.12721664	0.15152470	0.16634114	0.17711449
10 BG 5/6	0.07643438	0.08331559	0.11100201	0.14350983	0.17423900	0.19421370	0.20650217
2.5 B 5/6	0.08754887	0.08912258	0.11278367	0.14475281	0.17931493	0.20687443	0.22250149
5 B 5/6	0.09866464	0.10576750	0.11646948	0.14761381	0.18312129	0.21422115	0.23507806
7.5 B 5/6	0.10856559	0.11800558	0.12744557	0.15826258	0.19435325	0.22963026	0.25539447
10 B 5/6	0.12082020	0.12780114	0.13478209	0.16546128	0.20339451	0.24150542	0.27112223
2.5 PB 5/6	0.13444972	0.14532814	0.15620656	0.19681885	0.24251791	0.28147637	0.30614585
5 PB 5/6	0.14881474	0.15776074	0.16755051	0.21066340	0.25865584	0.29905690	0.32321493
7.5 PB 5/6	0.16539494	0.17078358	0.19966883	0.24192886	0.28435826	0.31782386	0.33609602
10 PB 5/6	0.16960801	0.17225306	0.17626260	0.21588880	0.26277293	0.30495553	0.33082564
2.5 P 5/6	0.17090173	0.18586544	0.20082915	0.21837811	0.27307909	0.31862133	0.34394805
5 P 5/6	0.16883181	0.17169571	0.17746089	0.22567665	0.28009908	0.32353210	0.34819688
7.5 P 5/6	0.15518821	0.16178615	0.19634560	0.24017960	0.28140116	0.30389812	0.31507058
10 P 5/6	0.14562743	0.15348406	0.16458616	0.20485425	0.24624104	0.27753054	0.29058925
2.5 RP 5/6	0.14416443	0.14697052	0.16793065	0.19965425	0.23297369	0.25586286	0.26544773
5 RP 5/6	0.13570760	0.14359834	0.15394712	0.18338017	0.20929293	0.22417187	0.22956708
7.5 RP 5/6	0.13844450	0.14430419	0.15735700	0.17978172	0.19706144	0.20564023	0.20720224
10 RP 5/6	0.13780126	0.14039940	0.15236103	0.16856738	0.17931097	0.18328333	0.18278427
3.75 Y 8/14	0.01720735	0.01884178	0.02047598	0.02210994	0.02374369	0.02582584	0.02508554

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	415nm	420nm	425nm	430nm	435nm	440nm	445nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.16545171	0.16396405	0.16190403	0.16078064	0.16019617	0.15970694	0.15830057
5 R 5/6	0.14688429	0.14568698	0.14411159	0.14304781	0.14244978	0.14183000	0.14079746
7.5 R 5/6	0.12930108	0.12791086	0.12665470	0.12566224	0.12518538	0.12479502	0.12402800
10 R 5/6	0.10603221	0.10556962	0.10540705	0.10491388	0.10462333	0.10437294	0.10404557
2.5 YR 5/6	0.08973322	0.08988318	0.09018108	0.09017446	0.08998800	0.08995148	0.08988984
5 YR 5/6	0.07606142	0.07591224	0.07583941	0.07599782	0.07587841	0.07613821	0.07617521
7.5 YR 5/6	0.06532461	0.06534611	0.06542858	0.06566564	0.06569151	0.06596174	0.06616417
10 YR 5/6	0.05579939	0.05566018	0.05566029	0.05578765	0.05593099	0.05601094	0.05627804
2.5 Y 5/6	0.04748886	0.04718929	0.04690048	0.04686503	0.04681075	0.04684346	0.04688840
5 Y 5/6	0.04152442	0.04106106	0.04043426	0.04020202	0.03989173	0.03987698	0.03963382
7.5 Y 5/6	0.03798544	0.03771861	0.03718935	0.03684351	0.03672678	0.03695728	0.03698148
10 Y 5/6	0.03904938	0.03858596	0.03846060	0.03819962	0.03818833	0.03851060	0.03871671
2.5 GY 5/6	0.04403860	0.04396146	0.04400334	0.04410966	0.04447710	0.04481730	0.04527048
5 GY 5/6	0.04947199	0.04957833	0.04955606	0.04970268	0.05005076	0.05042331	0.05123629
7.5 GY 5/6	0.06337902	0.06362642	0.06384795	0.06413782	0.06470537	0.06543337	0.06665364
10 GY 5/6	0.07946739	0.07989220	0.08044237	0.08091362	0.08170854	0.08285768	0.08446507
2.5 G 5/6	0.09005956	0.09140278	0.09288924	0.09457888	0.09624449	0.09939060	0.10249110
5 G 5/6	0.11131118	0.11321298	0.11443696	0.11620651	0.11846873	0.12095958	0.12428054
7.5 G 5/6	0.10932904	0.11206885	0.11515414	0.11869200	0.12348078	0.12822571	0.13479796
10 G 5/6	0.13107317	0.13351248	0.13662740	0.14039732	0.14474471	0.14863158	0.15475021
2.5 BG 5/6	0.14242949	0.14547734	0.14952576	0.15317733	0.15786350	0.16236980	0.16798679
5 BG 5/6	0.16362696	0.16767722	0.17269636	0.17854621	0.18298115	0.18893420	0.19610807
7.5 BG 5/6	0.18243419	0.18738764	0.19365485	0.19944201	0.20642630	0.21210963	0.22004829
10 BG 5/6	0.21257920	0.21804354	0.22466888	0.23027927	0.23763965	0.24476160	0.25194826
2.5 B 5/6	0.23112554	0.23490591	0.24380667	0.24731128	0.25651701	0.26412457	0.27328750
5 B 5/6	0.24554955	0.24948541	0.26195524	0.26651422	0.27715935	0.28966901	0.30068790
7.5 B 5/6	0.26732011	0.27072116	0.28534503	0.28936709	0.30056966	0.31599006	0.32686640
10 B 5/6	0.28511077	0.28992587	0.30306767	0.30677162	0.31704135	0.33188546	0.34395650
2.5 PB 5/6	0.31910168	0.32477411	0.32985158	0.33539075	0.34001804	0.34798135	0.35491888
5 PB 5/6	0.33466601	0.34046914	0.34477845	0.34910868	0.35210129	0.35681325	0.36209401
7.5 PB 5/6	0.34520040	0.35069647	0.35480333	0.35965333	0.36235230	0.36525843	0.36642243
10 PB 5/6	0.34420473	0.35127464	0.35712505	0.36080341	0.36377539	0.36509446	0.36384085
2.5 P 5/6	0.35463621	0.35915885	0.36060913	0.36097722	0.35810315	0.35492006	0.34921526
5 P 5/6	0.35663128	0.35895965	0.35667932	0.35277652	0.34774172	0.34167135	0.33297177
7.5 P 5/6	0.31717244	0.31737415	0.31621290	0.31444488	0.31181826	0.30889659	0.30421751
10 P 5/6	0.29590907	0.29669071	0.29531674	0.29226614	0.28844581	0.28234297	0.27805903
2.5 RP 5/6	0.26883611	0.26903133	0.26779894	0.26513107	0.26152996	0.25695493	0.25263137
5 RP 5/6	0.23046577	0.22954814	0.22826510	0.22560829	0.22344211	0.22083222	0.21820503
7.5 RP 5/6	0.20627576	0.20348673	0.20094478	0.19870275	0.19682302	0.19672075	0.19474103
10 RP 5/6	0.18105277	0.17880689	0.17646070	0.17529254	0.17462830	0.17506295	0.17370673
3.75 Y 8/14	0.02308791	0.02160289	0.01975342	0.01916607	0.01837189	0.01838242	0.01787978

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	450nm	455nm	460nm	465nm	470nm	475nm	480nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.15744392	0.15612129	0.15504438	0.15411405	0.15310764	0.15151599	0.14891914
5 R 5/6	0.14015551	0.13899435	0.13801157	0.13714923	0.13622648	0.13490883	0.13283917
7.5 R 5/6	0.12359121	0.12305296	0.12279151	0.12273842	0.12253807	0.12186670	0.12054299
10 R 5/6	0.10428166	0.10407493	0.10480031	0.10494445	0.10527772	0.10497434	0.10443269
2.5 YR 5/6	0.09042372	0.09045023	0.09126544	0.09123552	0.09150732	0.09131989	0.09118153
5 YR 5/6	0.07644696	0.07678135	0.07709733	0.07717334	0.07724843	0.07733151	0.07735088
7.5 YR 5/6	0.06642869	0.06667079	0.06708553	0.06720555	0.06730930	0.06757252	0.06764665
10 YR 5/6	0.05669596	0.05682358	0.05722843	0.05739123	0.05746054	0.05758176	0.05790403
2.5 Y 5/6	0.04732663	0.04757283	0.04809173	0.04858049	0.04945806	0.05079515	0.05310676
5 Y 5/6	0.04004614	0.04042669	0.04104196	0.04188933	0.04371166	0.04651524	0.05126198
7.5 Y 5/6	0.03733031	0.03779669	0.03849000	0.03960857	0.04167644	0.04507141	0.05055555
10 Y 5/6	0.03915862	0.04000166	0.04108526	0.04254269	0.04481186	0.04855270	0.05389431
2.5 GY 5/6	0.04577831	0.04649604	0.04737987	0.04855747	0.05029320	0.05296162	0.05703723
5 GY 5/6	0.05221380	0.05374195	0.05566247	0.05822124	0.06163384	0.06633566	0.07274366
7.5 GY 5/6	0.06810442	0.07031488	0.07317032	0.07687789	0.08192715	0.08851404	0.09731088
10 GY 5/6	0.08637878	0.08922714	0.09299579	0.09780276	0.10443164	0.11273686	0.12357168
2.5 G 5/6	0.10682043	0.11279120	0.12102847	0.13162106	0.14533737	0.16229354	0.18058961
5 G 5/6	0.12857183	0.13383003	0.14191838	0.15195849	0.16657167	0.18412983	0.20309902
7.5 G 5/6	0.14250202	0.15212480	0.16531277	0.18013107	0.19762036	0.21554169	0.23385645
10 G 5/6	0.16193877	0.17067048	0.18271079	0.19677570	0.21087256	0.22539072	0.23963546
2.5 BG 5/6	0.17730439	0.18712147	0.20052330	0.21847235	0.23718450	0.25012842	0.26798267
5 BG 5/6	0.20575134	0.21701884	0.23128104	0.24868499	0.26394734	0.27589436	0.28736597
7.5 BG 5/6	0.22909284	0.24093207	0.25541249	0.27336069	0.28835178	0.29854497	0.30540462
10 BG 5/6	0.26271552	0.27216901	0.28487331	0.29918220	0.31108313	0.31868074	0.32597510
2.5 B 5/6	0.28343677	0.29340816	0.30676384	0.32302123	0.33559925	0.34624009	0.35494512
5 B 5/6	0.31120852	0.32098776	0.33263201	0.34561529	0.35628493	0.36363673	0.36679811
7.5 B 5/6	0.33559823	0.34205726	0.34753088	0.35551343	0.35979280	0.36211196	0.36163120
10 B 5/6	0.35166437	0.35591469	0.35911719	0.36117555	0.36073735	0.35925096	0.35470965
2.5 PB 5/6	0.35963307	0.36048087	0.36123939	0.35964537	0.35674179	0.35288441	0.34793130
5 PB 5/6	0.36413517	0.36168936	0.36129423	0.35958580	0.35476466	0.34902804	0.34017419
7.5 PB 5/6	0.36503513	0.36193595	0.35782521	0.35221257	0.34534449	0.33649922	0.32569315
10 PB 5/6	0.35963069	0.35467945	0.34849053	0.34086934	0.33161055	0.32083899	0.30756075
2.5 P 5/6	0.34295607	0.33474152	0.32548509	0.31541000	0.30516413	0.29388093	0.28199901
5 P 5/6	0.32389182	0.31307127	0.30363810	0.29113657	0.27922710	0.26783403	0.25697143
7.5 P 5/6	0.29843704	0.29277261	0.28460362	0.27579218	0.26629186	0.25641137	0.24671459
10 P 5/6	0.27186617	0.26496155	0.25908423	0.25241511	0.24682393	0.23939544	0.23109419
2.5 RP 5/6	0.24672818	0.24146278	0.23558611	0.22984969	0.22421839	0.21800238	0.21103641
5 RP 5/6	0.21496414	0.21174558	0.20863454	0.20564104	0.20228158	0.19870850	0.19360897
7.5 RP 5/6	0.19272556	0.19040407	0.18875420	0.18746821	0.18596822	0.18342100	0.17914825
10 RP 5/6	0.17280332	0.17189907	0.17136578	0.17107855	0.17054594	0.16889117	0.16598683
3.75 Y 8/14	0.01943612	0.02065597	0.02279359	0.02542845	0.03085745	0.03919190	0.05336837

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	485nm	490nm	495nm	500nm	505nm	510nm	515nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.14555931	0.14190003	0.13915852	0.13741133	0.13580306	0.13497738	0.13371820
5 R 5/6	0.13055595	0.12802930	0.12616341	0.12475428	0.12358656	0.12301672	0.12231167
7.5 R 5/6	0.11908171	0.11765798	0.11702508	0.11634849	0.11592052	0.11551840	0.11452513
10 R 5/6	0.10454436	0.10472618	0.10574915	0.10629851	0.10794620	0.11069334	0.11367771
2.5 YR 5/6	0.09201579	0.09286765	0.09421968	0.09575183	0.09925993	0.10569313	0.11374810
5 YR 5/6	0.07793419	0.07861274	0.07940808	0.08127388	0.08527625	0.09324275	0.10427488
7.5 YR 5/6	0.06828322	0.06901981	0.06994712	0.07176728	0.07578100	0.08442179	0.09798332
10 YR 5/6	0.05828402	0.05917598	0.06040373	0.06367659	0.07064106	0.08419215	0.10386791
2.5 Y 5/6	0.05642666	0.06170890	0.06858389	0.07831844	0.09127407	0.10907270	0.12928324
5 Y 5/6	0.05845089	0.06977465	0.08493103	0.10370380	0.12314063	0.14215452	0.15793737
7.5 Y 5/6	0.05929293	0.07205919	0.08919678	0.11045830	0.13297483	0.15458253	0.17285209
10 Y 5/6	0.06162786	0.07254910	0.08533887	0.10024768	0.11504577	0.13094755	0.14852500
2.5 GY 5/6	0.06322362	0.07283942	0.08615452	0.10389215	0.12521210	0.14818231	0.17036708
5 GY 5/6	0.08143850	0.09394081	0.10980260	0.12984123	0.15242780	0.17603042	0.19859010
7.5 GY 5/6	0.10845976	0.12357343	0.14178362	0.16352721	0.18639732	0.20882176	0.22864152
10 GY 5/6	0.13658104	0.15321834	0.17236308	0.19405279	0.21543976	0.23515304	0.25123088
2.5 G 5/6	0.20179365	0.22131862	0.23904194	0.25259155	0.26148741	0.26560924	0.26785855
5 G 5/6	0.22064050	0.23868983	0.25379550	0.26563257	0.27397762	0.27792941	0.27874336
7.5 G 5/6	0.25009686	0.26529501	0.27465650	0.27996791	0.28194870	0.28180805	0.27937821
10 G 5/6	0.25416903	0.26763611	0.27742874	0.28365423	0.28621186	0.28707201	0.28529188
2.5 BG 5/6	0.28139865	0.28870211	0.29711028	0.29812391	0.29631546	0.29254606	0.28816683
5 BG 5/6	0.29526659	0.30005081	0.30428737	0.30411084	0.30099955	0.29734091	0.29307741
7.5 BG 5/6	0.31196231	0.31558418	0.31664047	0.31611010	0.31252111	0.30765523	0.30198165
10 BG 5/6	0.32952713	0.33198562	0.33150067	0.32883917	0.32358778	0.31675605	0.31015359
2.5 B 5/6	0.35949086	0.35936364	0.35745078	0.35218047	0.34413378	0.33390211	0.32035405
5 B 5/6	0.36893110	0.36717826	0.36239116	0.35448064	0.34446241	0.33138747	0.31654469
7.5 B 5/6	0.35893784	0.35468486	0.34887046	0.34103767	0.33071152	0.31862662	0.30475938
10 B 5/6	0.34924623	0.34325251	0.33572010	0.32662805	0.31619328	0.30407021	0.29144628
2.5 PB 5/6	0.33980266	0.33083339	0.32266440	0.31227546	0.30177427	0.28977037	0.27747540
5 PB 5/6	0.32917270	0.31549240	0.30218623	0.28965691	0.27829847	0.26676274	0.25250463
7.5 PB 5/6	0.31155944	0.29584067	0.28052623	0.26746379	0.25504881	0.24294783	0.23044736
10 PB 5/6	0.29121795	0.27437976	0.25874782	0.24529845	0.23402046	0.22305516	0.21169838
2.5 P 5/6	0.26837450	0.25511160	0.24125862	0.22922777	0.21962158	0.21078932	0.20284463
5 P 5/6	0.24587579	0.23414607	0.22300474	0.21288104	0.20502791	0.19889529	0.19368631
7.5 P 5/6	0.23694859	0.22777036	0.21835840	0.20990510	0.20084239	0.19368748	0.18660195
10 P 5/6	0.22166727	0.21262307	0.20529161	0.19967306	0.19383070	0.18770303	0.17890006
2.5 RP 5/6	0.20345847	0.19685410	0.19112116	0.18622567	0.18182696	0.17677933	0.16975725
5 RP 5/6	0.18800235	0.18290802	0.17846929	0.17485387	0.17122517	0.16698524	0.16079552
7.5 RP 5/6	0.17343106	0.16808892	0.16403313	0.16129044	0.15938654	0.15681610	0.15281145
10 RP 5/6	0.16161807	0.15724776	0.15418984	0.15228895	0.15061434	0.14900619	0.14604835
3.75 Y 8/14	0.07459083	0.10809208	0.15266520	0.20954208	0.27236778	0.34083996	0.40511648

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	520nm	525nm	530nm	535nm	540nm	545nm	550nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.13181078	0.12994925	0.12853077	0.12761498	0.12787532	0.12960040	0.13309883
5 R 5/6	0.12110696	0.11999334	0.11926960	0.11891307	0.11991005	0.12224048	0.12765039
7.5 R 5/6	0.11320462	0.11247507	0.11272791	0.11380942	0.11657011	0.12126172	0.13054378
10 R 5/6	0.11539499	0.11663225	0.11801447	0.11934692	0.12256201	0.12688786	0.13484854
2.5 YR 5/6	0.12019203	0.12448144	0.12737217	0.12927072	0.13247919	0.13617406	0.14265206
5 YR 5/6	0.11532051	0.12353463	0.12875922	0.13250066	0.13678865	0.14306870	0.15434399
7.5 YR 5/6	0.11302328	0.12485609	0.13170265	0.13584677	0.14061977	0.14925650	0.16498146
10 YR 5/6	0.12288699	0.13576397	0.14212918	0.14566417	0.15104130	0.16149451	0.18097304
2.5 Y 5/6	0.14623756	0.15721251	0.16279057	0.16608738	0.17033811	0.17766298	0.19120874
5 Y 5/6	0.16932712	0.17683631	0.18111014	0.18395746	0.18662968	0.19007189	0.19696957
7.5 Y 5/6	0.18634227	0.19451659	0.19995202	0.20333069	0.20562723	0.20739383	0.20950366
10 Y 5/6	0.16885940	0.19044685	0.20884090	0.22085243	0.22776892	0.23159907	0.23293866
2.5 GY 5/6	0.18943975	0.20528854	0.21725364	0.22688465	0.23365082	0.23863812	0.24022018
5 GY 5/6	0.21659760	0.22938316	0.23697140	0.24038428	0.24151884	0.24136548	0.23965078
7.5 GY 5/6	0.24288404	0.25140378	0.25504973	0.25468223	0.25200129	0.24764278	0.24185285
10 GY 5/6	0.26133617	0.26574919	0.26595232	0.26249541	0.25688788	0.24942466	0.24072646
2.5 G 5/6	0.26695694	0.26373655	0.26077448	0.25514067	0.24955505	0.24245209	0.23360972
5 G 5/6	0.27744952	0.27286173	0.26732926	0.26175660	0.25362146	0.24421858	0.23407110
7.5 G 5/6	0.27583515	0.27093597	0.26561247	0.25954669	0.25158960	0.24214641	0.23146215
10 G 5/6	0.28185847	0.27728790	0.27133367	0.26335749	0.25550801	0.24461183	0.23262820
2.5 BG 5/6	0.28351986	0.27713397	0.27021469	0.26172779	0.25205811	0.24079934	0.22859182
5 BG 5/6	0.28759079	0.28056332	0.27283733	0.26306449	0.25294395	0.24144556	0.22856867
7.5 BG 5/6	0.29504508	0.28682898	0.27711096	0.26513221	0.25017835	0.23653019	0.22248515
10 BG 5/6	0.30078728	0.29008155	0.27813727	0.26389423	0.25061309	0.23488143	0.21738641
2.5 B 5/6	0.30582444	0.29037437	0.27296442	0.25584803	0.23745676	0.22060660	0.20266951
5 B 5/6	0.30050525	0.28294786	0.26537783	0.24709813	0.23002610	0.21273594	0.19579284
7.5 B 5/6	0.28983692	0.27356912	0.25788233	0.24116190	0.22597839	0.20976022	0.19456625
10 B 5/6	0.27758544	0.26252795	0.24847288	0.23305689	0.21929976	0.20531891	0.19159805
2.5 PB 5/6	0.26286741	0.24910426	0.23632504	0.22416844	0.21238710	0.20114384	0.18977261
5 PB 5/6	0.23976364	0.22862806	0.21928266	0.21037755	0.20337101	0.19655454	0.18912350
7.5 PB 5/6	0.21777697	0.20699081	0.19897448	0.19349752	0.18984251	0.18665483	0.18335018
10 PB 5/6	0.20021346	0.19109029	0.18432219	0.18081658	0.17883709	0.17839719	0.17741843
2.5 P 5/6	0.19422024	0.18538033	0.17884998	0.17481934	0.17282599	0.17304563	0.17400291
5 P 5/6	0.18786553	0.18120707	0.17550263	0.17123949	0.16941518	0.16962239	0.17170314
7.5 P 5/6	0.18005801	0.17326077	0.16764434	0.16334865	0.16081052	0.15952505	0.15958558
10 P 5/6	0.16797599	0.15788864	0.15012037	0.14595688	0.14562743	0.14825563	0.15171067
2.5 RP 5/6	0.16146834	0.15347240	0.14748110	0.14448261	0.14418902	0.14663161	0.14929867
5 RP 5/6	0.15317611	0.14502286	0.13883571	0.13570760	0.13599332	0.13885672	0.14281915
7.5 RP 5/6	0.14745279	0.14278286	0.13971088	0.13844450	0.13915480	0.14174169	0.14529147
10 RP 5/6	0.14254220	0.13969039	0.13814388	0.13780126	0.13869293	0.14128363	0.14475180
3.75 Y 8/14	0.45474433	0.48718594	0.50473540	0.51585326	0.52787033	0.54588087	0.58042190

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	555nm	560nm	565nm	570nm	575nm	580nm	585nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.13865206	0.14738198	0.15872583	0.17289904	0.19275551	0.21987714	0.25296009
5 R 5/6	0.13737623	0.15327503	0.17276866	0.19349237	0.21752162	0.24694952	0.27824821
7.5 R 5/6	0.14514540	0.16573296	0.18696457	0.20577236	0.22639752	0.25338002	0.28363055
10 R 5/6	0.14768143	0.16493203	0.18457485	0.20464105	0.22839235	0.25704501	0.28753106
2.5 YR 5/6	0.15270464	0.16677280	0.18578380	0.20873107	0.23537787	0.26301310	0.28846575
5 YR 5/6	0.17017771	0.19101485	0.21386748	0.23602098	0.25635187	0.27417792	0.28782969
7.5 YR 5/6	0.18785432	0.21488434	0.23975594	0.25768918	0.26946762	0.27733353	0.28246049
10 YR 5/6	0.20728990	0.23335335	0.25253718	0.26338176	0.26925712	0.27223810	0.27370286
2.5 Y 5/6	0.20777974	0.22471026	0.23776418	0.24675735	0.25230814	0.25586588	0.25801771
5 Y 5/6	0.20423285	0.21357799	0.22212100	0.23001904	0.23557854	0.23977954	0.24261026
7.5 Y 5/6	0.21204708	0.21550877	0.21809941	0.22108112	0.22232173	0.22391358	0.22480399
10 Y 5/6	0.23289574	0.23334475	0.23275716	0.23115792	0.22891083	0.22683012	0.22508205
2.5 GY 5/6	0.24128542	0.24035811	0.23864475	0.23536502	0.23095048	0.22655348	0.22061317
5 GY 5/6	0.23649328	0.23214418	0.22805853	0.22270137	0.21782581	0.21190692	0.20503484
7.5 GY 5/6	0.23487755	0.22702994	0.21910995	0.21044032	0.20212242	0.19388189	0.18481672
10 GY 5/6	0.23114146	0.22099272	0.21062079	0.19993647	0.18957272	0.18003987	0.16986695
2.5 G 5/6	0.22417363	0.21390760	0.20387408	0.19308725	0.18229607	0.17118168	0.16054010
5 G 5/6	0.22226873	0.21036467	0.19808729	0.18552480	0.17401316	0.16261479	0.15033484
7.5 G 5/6	0.22128043	0.20932859	0.19686114	0.18459229	0.17191495	0.15858506	0.14519809
10 G 5/6	0.22018939	0.20746276	0.19324349	0.17985949	0.16606852	0.15251592	0.13943013
2.5 BG 5/6	0.21541362	0.20273587	0.18951276	0.17534828	0.16167798	0.14918700	0.13620147
5 BG 5/6	0.21319614	0.19902362	0.18406858	0.17008172	0.15603550	0.14336755	0.13054685
7.5 BG 5/6	0.20795538	0.19091536	0.17647500	0.16254722	0.14914326	0.13732253	0.12504871
10 BG 5/6	0.19939610	0.18225976	0.16580135	0.15145603	0.13905955	0.12861594	0.11938679
2.5 B 5/6	0.18521879	0.16821204	0.15470799	0.14236314	0.13299106	0.12510069	0.11847556
5 B 5/6	0.17960710	0.16461473	0.15190480	0.14114324	0.13274019	0.12600722	0.12089607
7.5 B 5/6	0.17949336	0.16572674	0.15431645	0.14461029	0.13716224	0.13116341	0.12671257
10 B 5/6	0.17850455	0.16658323	0.15646460	0.14862423	0.14271130	0.13827465	0.13492008
2.5 PB 5/6	0.17754281	0.16628895	0.15709732	0.15075645	0.14655090	0.14393050	0.14278050
5 PB 5/6	0.18095778	0.17199481	0.16488178	0.15985272	0.15659230	0.15545077	0.15528566
7.5 PB 5/6	0.17883065	0.17359041	0.16920860	0.16659736	0.16679289	0.16841034	0.17130454
10 PB 5/6	0.17550412	0.17224551	0.16972831	0.16990035	0.17173308	0.17622697	0.18278204
2.5 P 5/6	0.17424318	0.17317397	0.17166755	0.17092329	0.17223776	0.17538497	0.18016523
5 P 5/6	0.17367603	0.17505347	0.17466317	0.17311218	0.17170651	0.17223300	0.17383824
7.5 P 5/6	0.15909303	0.15783073	0.15647712	0.15702307	0.16254557	0.17419136	0.18728428
10 P 5/6	0.15305759	0.15174345	0.15185815	0.15549423	0.16566491	0.18318363	0.20698116
2.5 RP 5/6	0.15070535	0.15036174	0.15086695	0.15450283	0.16441346	0.18072147	0.20398804
5 RP 5/6	0.14552391	0.14630964	0.14820855	0.15448007	0.16788818	0.18884978	0.21695479
7.5 RP 5/6	0.14814678	0.14856029	0.14838665	0.15060610	0.15943256	0.17856294	0.20861892
10 RP 5/6	0.14733597	0.14772138	0.14674564	0.14764356	0.15648810	0.17754311	0.21334797
3.75 Y 8/14	0.61995126	0.66480282	0.70248812	0.73311979	0.75361484	0.76823047	0.77774289

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	590nm	595nm	600nm	605nm	610nm	615nm	620nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.28613051	0.31514409	0.33860005	0.35578339	0.36607711	0.37278382	0.37823205
5 R 5/6	0.30570601	0.32732855	0.34299353	0.35383150	0.35949938	0.36343271	0.36689793
7.5 R 5/6	0.30971144	0.33003960	0.34358032	0.35265525	0.35699357	0.35945087	0.36181124
10 R 5/6	0.31467796	0.33501022	0.34919398	0.35630686	0.35917611	0.35830702	0.35778633
2.5 YR 5/6	0.31131929	0.32771438	0.33937867	0.34484431	0.34752305	0.34646523	0.34516198
5 YR 5/6	0.29923299	0.30737584	0.31207900	0.31563152	0.31853298	0.31994862	0.32034217
7.5 YR 5/6	0.28653955	0.29024100	0.29209239	0.29402732	0.29607896	0.29707550	0.29766649
10 YR 5/6	0.27451371	0.27525422	0.27440594	0.27506755	0.27498092	0.27443001	0.27388633
2.5 Y 5/6	0.25940841	0.26038328	0.25982854	0.26035748	0.25988621	0.25999415	0.25928123
5 Y 5/6	0.24458800	0.24587821	0.24568094	0.24616798	0.24551161	0.24619544	0.24541329
7.5 Y 5/6	0.22592985	0.22575652	0.22640850	0.22725937	0.22711436	0.22762858	0.22805620
10 Y 5/6	0.22157947	0.21829268	0.21289135	0.20815373	0.20278478	0.19863246	0.19522900
2.5 GY 5/6	0.21340179	0.20473046	0.19540238	0.18519977	0.17660468	0.16944696	0.16357823
5 GY 5/6	0.19675883	0.18816813	0.17826937	0.16853064	0.16040232	0.15315673	0.14768234
7.5 GY 5/6	0.17513791	0.16510371	0.15481784	0.14515906	0.13725709	0.13070656	0.12523057
10 GY 5/6	0.15959222	0.14902103	0.13885407	0.12959027	0.12210986	0.11619900	0.11095895
2.5 G 5/6	0.14843864	0.13716516	0.12599567	0.11607887	0.10787498	0.10141993	0.09636708
5 G 5/6	0.13839685	0.12687295	0.11591273	0.10611399	0.09820977	0.09223963	0.08803078
7.5 G 5/6	0.13216980	0.11936379	0.10802878	0.09924148	0.09241924	0.08783515	0.08438779
10 G 5/6	0.12590588	0.11319129	0.10301867	0.09452527	0.08798505	0.08364023	0.08029693
2.5 BG 5/6	0.12271288	0.11098453	0.10058728	0.09270114	0.08686902	0.08257811	0.07946316
5 BG 5/6	0.11858694	0.10730451	0.09621224	0.08883670	0.08344091	0.07933235	0.07624038
7.5 BG 5/6	0.11448877	0.10410936	0.09476004	0.08778026	0.08215695	0.07824045	0.07628129
10 BG 5/6	0.11091776	0.10315071	0.09580321	0.08952295	0.08485981	0.08221319	0.08047648
2.5 B 5/6	0.11236552	0.10696344	0.10154430	0.09766289	0.09396173	0.09220327	0.09072260
5 B 5/6	0.11614364	0.11200842	0.10813340	0.10485555	0.10237309	0.10078579	0.09977654
7.5 B 5/6	0.12287507	0.11947653	0.11623878	0.11346074	0.11129668	0.10989977	0.10919557
10 B 5/6	0.13199498	0.12925597	0.12644021	0.12412066	0.12239785	0.12131656	0.12092468
2.5 PB 5/6	0.14161109	0.13999070	0.13813235	0.13633912	0.13509790	0.13447875	0.13488184
5 PB 5/6	0.15542014	0.15549970	0.15504478	0.15311976	0.15142547	0.14969121	0.14906045
7.5 PB 5/6	0.17413028	0.17723560	0.17791937	0.17756246	0.17606011	0.17354805	0.17223735
10 PB 5/6	0.19044056	0.19773221	0.20254312	0.20413966	0.20269397	0.19858126	0.19360639
2.5 P 5/6	0.18700787	0.19661050	0.20672195	0.21668724	0.22409063	0.22620927	0.22569282
5 P 5/6	0.17876984	0.18776059	0.20118779	0.21889555	0.23776397	0.25410482	0.26677945
7.5 P 5/6	0.19993588	0.21432488	0.23105144	0.25111101	0.27259344	0.29106187	0.30768630
10 P 5/6	0.23288914	0.25765225	0.27882342	0.29501904	0.30658073	0.31504155	0.32100140
2.5 RP 5/6	0.23128983	0.25936260	0.28514745	0.30642515	0.32432419	0.33820179	0.34717293
5 RP 5/6	0.24821835	0.27831277	0.30579181	0.32697156	0.34477533	0.35612314	0.36496712
7.5 RP 5/6	0.24499274	0.28138366	0.31471777	0.34113680	0.36109786	0.37579287	0.38613153
10 RP 5/6	0.25622285	0.29801237	0.33335378	0.36009196	0.37709515	0.38728412	0.39463545
3.75 Y 8/14	0.78423467	0.78856225	0.78734539	0.78919924	0.78703144	0.78876571	0.78597540

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	625nm	630nm	635nm	640nm	645nm	650nm	655nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.37938811	0.38084456	0.38101363	0.38190469	0.38139059	0.38163536	0.38217105
5 R 5/6	0.36684945	0.36710972	0.36689738	0.36802363	0.36715131	0.36737358	0.36729520
7.5 R 5/6	0.36135201	0.36092928	0.36050123	0.36152681	0.36061416	0.36124622	0.36008770
10 R 5/6	0.35585020	0.35410380	0.35186541	0.34981134	0.34838637	0.34597545	0.34498085
2.5 YR 5/6	0.34382007	0.34255687	0.34049101	0.33762293	0.33675039	0.33388551	0.33347891
5 YR 5/6	0.32136648	0.32231103	0.32254998	0.32242625	0.32326759	0.32379041	0.32347245
7.5 YR 5/6	0.29874771	0.30042400	0.30080018	0.30123940	0.30262043	0.30323655	0.30346640
10 YR 5/6	0.27346409	0.27303409	0.27295466	0.27325514	0.27259693	0.27209696	0.27220847
2.5 Y 5/6	0.25924252	0.25862529	0.25854808	0.25890984	0.25807024	0.25783263	0.25790350
5 Y 5/6	0.24574111	0.24508778	0.24503506	0.24544829	0.24457510	0.24459456	0.24463967
7.5 Y 5/6	0.22828104	0.22882989	0.22827634	0.22944260	0.22893061	0.22867459	0.22997033
10 Y 5/6	0.19189050	0.18931215	0.18651587	0.18425925	0.18172737	0.17918445	0.17769027
2.5 GY 5/6	0.15890493	0.15569904	0.15193187	0.14913088	0.14621228	0.14296772	0.14062850
5 GY 5/6	0.14337828	0.13998075	0.13653149	0.13357645	0.13047307	0.12777822	0.12572562
7.5 GY 5/6	0.12139128	0.11841297	0.11547744	0.11278899	0.11005648	0.10776878	0.10606196
10 GY 5/6	0.10752996	0.10489749	0.10234078	0.09992395	0.09750531	0.09551906	0.09407779
2.5 G 5/6	0.09258158	0.08950413	0.08588745	0.08375607	0.08100976	0.07826755	0.07692296
5 G 5/6	0.08405135	0.08193070	0.07907497	0.07762373	0.07542297	0.07387014	0.07252307
7.5 G 5/6	0.08114428	0.07993411	0.07760179	0.07600575	0.07445754	0.07388136	0.07281307
10 G 5/6	0.07858957	0.07676667	0.07506848	0.07399049	0.07246335	0.07149829	0.07171876
2.5 BG 5/6	0.07720312	0.07582507	0.07404340	0.07285565	0.07203912	0.07061577	0.07113371
5 BG 5/6	0.07379066	0.07304403	0.07121733	0.07037286	0.06911816	0.06805480	0.06788208
7.5 BG 5/6	0.07366885	0.07294250	0.07139763	0.07182384	0.07090257	0.07054756	0.07190650
10 BG 5/6	0.07855228	0.07770042	0.07662048	0.07694688	0.07694681	0.07646802	0.07843863
2.5 B 5/6	0.08998730	0.08864307	0.08820634	0.08756111	0.08768621	0.08821355	0.08914824
5 B 5/6	0.09965224	0.09929540	0.09877177	0.09929649	0.09991702	0.10064657	0.10204525
7.5 B 5/6	0.10902456	0.10868446	0.10872610	0.10912487	0.10977467	0.11067536	0.11224680
10 B 5/6	0.12118464	0.12175536	0.12225183	0.12328220	0.12463337	0.12595147	0.12779808
2.5 PB 5/6	0.13676999	0.13992230	0.14306905	0.14718265	0.15148005	0.15545503	0.15860869
5 PB 5/6	0.15021212	0.15255946	0.15616304	0.16081479	0.16636766	0.17208826	0.17745090
7.5 PB 5/6	0.17153980	0.17408762	0.17793737	0.18439742	0.19195693	0.20000901	0.20871952
10 PB 5/6	0.19113040	0.19250737	0.19790919	0.20645985	0.21801732	0.23169689	0.24733896
2.5 P 5/6	0.22556626	0.22872702	0.23624636	0.24691250	0.26122215	0.27662488	0.29424103
5 P 5/6	0.27513420	0.28475863	0.29401487	0.30412978	0.31468053	0.32517707	0.33507274
7.5 P 5/6	0.32057615	0.32951276	0.33452905	0.33926109	0.34166446	0.34460698	0.34472530
10 P 5/6	0.32435217	0.32809240	0.32948953	0.33017426	0.33083124	0.33072826	0.33154458
2.5 RP 5/6	0.35365681	0.35958515	0.36292766	0.36505704	0.36651525	0.36735067	0.36892962
5 RP 5/6	0.37108038	0.37619269	0.37972800	0.38250601	0.38430560	0.38417937	0.38714364
7.5 RP 5/6	0.39265378	0.39853536	0.40108677	0.40392484	0.40563710	0.40521479	0.40706869
10 RP 5/6	0.39751521	0.40025010	0.40071714	0.40126780	0.40139383	0.40149295	0.40190932
3.75 Y 8/14	0.78668298	0.78432252	0.78409453	0.78554836	0.78237344	0.78209340	0.78229506

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	660nm	665nm	670nm	675nm	680nm	685nm	690nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.38207299	0.38228419	0.38250026	0.38136232	0.38176173	0.38184695	0.38198269
5 R 5/6	0.36757409	0.36756337	0.36772748	0.36632829	0.36651137	0.36664246	0.36629473
7.5 R 5/6	0.36011713	0.36021371	0.35984693	0.35851688	0.35862308	0.35872287	0.35838048
10 R 5/6	0.34355655	0.34332887	0.34403869	0.34463443	0.34560900	0.34693936	0.34993008
2.5 YR 5/6	0.33218306	0.33118061	0.33282065	0.33354380	0.33496573	0.33593320	0.33914721
5 YR 5/6	0.32429840	0.32347765	0.32424057	0.32391575	0.32475001	0.32441615	0.32455944
7.5 YR 5/6	0.30486079	0.30555490	0.30520137	0.30672256	0.30716743	0.30804821	0.30922718
10 YR 5/6	0.27141752	0.27176163	0.27016202	0.27025147	0.26942364	0.26928575	0.26989790
2.5 Y 5/6	0.25710834	0.25721678	0.25608887	0.25603706	0.25520625	0.25519422	0.25571003
5 Y 5/6	0.24395359	0.24389796	0.24320025	0.24311076	0.24234558	0.24248077	0.24294719
7.5 Y 5/6	0.23001917	0.23088650	0.23098487	0.23140643	0.23194642	0.23205685	0.23282953
10 Y 5/6	0.17625476	0.17559400	0.17622615	0.17598355	0.17693485	0.17812791	0.18016804
2.5 GY 5/6	0.13895356	0.13839727	0.13832188	0.13902578	0.14060511	0.14321695	0.14575751
5 GY 5/6	0.12389214	0.12264344	0.12202315	0.12260927	0.12317599	0.12416643	0.12580107
7.5 GY 5/6	0.10466367	0.10364718	0.10303045	0.10328194	0.10360634	0.10424844	0.10541505
10 GY 5/6	0.09296852	0.09211152	0.09151873	0.09155820	0.09173890	0.09216151	0.09302402
2.5 G 5/6	0.07554678	0.07428564	0.07345377	0.07265424	0.07282260	0.07304385	0.07382440
5 G 5/6	0.07130481	0.07057848	0.07002575	0.06987039	0.07025913	0.07044024	0.07126675
7.5 G 5/6	0.07272408	0.07365263	0.07496374	0.07608345	0.07898720	0.08073055	0.08216705
10 G 5/6	0.07217791	0.07365623	0.07591905	0.07754205	0.07985311	0.08264579	0.08514544
2.5 BG 5/6	0.07147484	0.07330910	0.07447605	0.07731506	0.07900311	0.08183018	0.08422631
5 BG 5/6	0.06882844	0.07008789	0.07171002	0.07347253	0.07564218	0.07801296	0.08001648
7.5 BG 5/6	0.07243316	0.07414956	0.07490598	0.07600806	0.07802235	0.07911085	0.08067270
10 BG 5/6	0.07895533	0.08017316	0.08134439	0.08176925	0.08227687	0.08232880	0.08280311
2.5 B 5/6	0.09012552	0.09113642	0.09167975	0.09136784	0.09116674	0.08995045	0.08933174
5 B 5/6	0.10351249	0.10454305	0.10512617	0.10439554	0.10359460	0.10238041	0.10164080
7.5 B 5/6	0.11389835	0.11475449	0.11475532	0.11402068	0.11288310	0.11166792	0.11059818
10 B 5/6	0.12951159	0.13023179	0.12964556	0.12888776	0.12697824	0.12519246	0.12335055
2.5 PB 5/6	0.16104684	0.16187328	0.16028117	0.15707093	0.15314243	0.14912712	0.14635080
5 PB 5/6	0.18186767	0.18363707	0.18398676	0.18076628	0.17747811	0.17311403	0.17057967
7.5 PB 5/6	0.21660788	0.22168620	0.22445959	0.22365978	0.22196623	0.21933638	0.21644740
10 PB 5/6	0.26359423	0.27763136	0.29025009	0.30009125	0.30603954	0.31064741	0.31576139
2.5 P 5/6	0.31192895	0.32430949	0.33607154	0.34593806	0.35410026	0.35926072	0.36365517
5 P 5/6	0.34461054	0.35075361	0.35464530	0.36000479	0.36262341	0.36469191	0.36681267
7.5 P 5/6	0.34594283	0.34614458	0.34504115	0.34413039	0.34526972	0.34504416	0.34508205
10 P 5/6	0.33144425	0.33053458	0.33049644	0.32980932	0.33028156	0.32913219	0.32831509
2.5 RP 5/6	0.36954461	0.36949042	0.36972599	0.36996713	0.36907692	0.36968676	0.36829259
5 RP 5/6	0.38887750	0.39004060	0.39302632	0.39283436	0.39378408	0.39475018	0.39804444
7.5 RP 5/6	0.40649531	0.40606859	0.40666380	0.40632309	0.40644293	0.40526019	0.40546714
10 RP 5/6	0.40033578	0.40087671	0.40056157	0.40021830	0.40058352	0.40032514	0.40096438
3.75 Y 8/14	0.77961507	0.77963442	0.77647209	0.77619356	0.77328127	0.77357847	0.77536650

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	695nm	700nm	705nm	710nm	715nm	720nm	725nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.38153772	0.38118354	0.38146585	0.38152068	0.38161797	0.38210313	0.38116666
5 R 5/6	0.36652064	0.36597238	0.36581500	0.36582002	0.36565165	0.36571515	0.36533320
7.5 R 5/6	0.35852119	0.35810297	0.35782269	0.35711982	0.35688262	0.35683008	0.35725356
10 R 5/6	0.35295660	0.35615863	0.35611639	0.35570910	0.35570254	0.35576857	0.35632108
2.5 YR 5/6	0.34245593	0.34649073	0.34634509	0.34627501	0.34678948	0.34643607	0.34669357
5 YR 5/6	0.32471275	0.32537664	0.32493642	0.32461550	0.32521748	0.32444922	0.32464548
7.5 YR 5/6	0.30918669	0.30911809	0.30909573	0.30912880	0.30912789	0.30910174	0.30911639
10 YR 5/6	0.26950314	0.26883423	0.26861620	0.26893871	0.26892980	0.26867485	0.26881767
2.5 Y 5/6	0.25526535	0.25425345	0.25461362	0.25448288	0.25474489	0.25490639	0.25465404
5 Y 5/6	0.24248433	0.24124457	0.24205722	0.24156400	0.24203323	0.24252625	0.24196431
7.5 Y 5/6	0.23277249	0.23261539	0.23272569	0.23265662	0.23271901	0.23278730	0.23271111
10 Y 5/6	0.18223816	0.18418229	0.18594278	0.18862808	0.18944494	0.18887383	0.18899870
2.5 GY 5/6	0.14964582	0.15366198	0.15808185	0.16010995	0.16229678	0.16265191	0.16286557
5 GY 5/6	0.12856065	0.13153830	0.13362158	0.13592269	0.13653803	0.13599107	0.13591408
7.5 GY 5/6	0.10735695	0.10922034	0.11112624	0.11258142	0.11322739	0.11325602	0.11301938
10 GY 5/6	0.09442273	0.09562130	0.09734495	0.09830303	0.09895617	0.09930053	0.09903163
2.5 G 5/6	0.07445205	0.07590167	0.07778534	0.07925907	0.08077797	0.08176695	0.08295116
5 G 5/6	0.07237739	0.07368059	0.07557336	0.07644201	0.07744740	0.07843031	0.07878234
7.5 G 5/6	0.08503320	0.08592070	0.08821703	0.08896296	0.08860776	0.08928863	0.08883230
10 G 5/6	0.08725481	0.08883540	0.09003390	0.09076438	0.08975728	0.08884976	0.08925282
2.5 BG 5/6	0.08629889	0.08875103	0.08904544	0.09001576	0.08897116	0.08856860	0.08810150
5 BG 5/6	0.08153733	0.08409724	0.08381877	0.08370750	0.08338215	0.08315133	0.08464561
7.5 BG 5/6	0.08105497	0.08173629	0.08110130	0.08158813	0.08189516	0.08258484	0.08392258
10 BG 5/6	0.08104503	0.08117867	0.08014518	0.07937810	0.08042044	0.08276214	0.08455156
2.5 B 5/6	0.08768620	0.08664659	0.08466538	0.08408418	0.08411787	0.08539718	0.08752706
5 B 5/6	0.09981865	0.09796959	0.09660415	0.09600593	0.09658733	0.09865134	0.10155096
7.5 B 5/6	0.10863969	0.10666028	0.10527952	0.10474666	0.10558042	0.10704411	0.11027088
10 B 5/6	0.12168716	0.12024777	0.11883089	0.11827962	0.11930738	0.12145600	0.12457131
2.5 PB 5/6	0.14453119	0.14444705	0.14507974	0.14645087	0.14824876	0.15056193	0.15335240
5 PB 5/6	0.16906088	0.16904053	0.16955580	0.17130321	0.17368461	0.17724262	0.17968215
7.5 PB 5/6	0.21634631	0.21602771	0.21844486	0.21973786	0.22274297	0.22520064	0.22976261
10 PB 5/6	0.31873054	0.32271648	0.32766796	0.33168922	0.33616261	0.34164445	0.34535124
2.5 P 5/6	0.36387390	0.36416927	0.36452435	0.36481623	0.36513438	0.36552983	0.36578478
5 P 5/6	0.36671427	0.36679992	0.36658708	0.36663527	0.36646493	0.36650432	0.36633130
7.5 P 5/6	0.34405712	0.34496500	0.34276069	0.34322769	0.34128153	0.34194994	0.34006747
10 P 5/6	0.32806219	0.32846780	0.32759167	0.32772355	0.32636631	0.32765982	0.32660797
2.5 RP 5/6	0.36885364	0.36899822	0.36804221	0.36871229	0.36896938	0.36876161	0.36841757
5 RP 5/6	0.39814331	0.39806690	0.39808209	0.39809727	0.39811246	0.39812765	0.39814283
7.5 RP 5/6	0.40540414	0.40485627	0.40517810	0.40451715	0.40433832	0.40466938	0.40368509
10 RP 5/6	0.39955745	0.39942979	0.40022650	0.39921307	0.39937113	0.40029326	0.39897485
3.75 Y 8/14	0.77368407	0.76942287	0.77180122	0.77047797	0.77192520	0.77329008	0.77164097

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	730nm	735nm	740nm	745nm	750nm	755nm	760nm
10N	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.38089866	0.38077363	0.38097794	0.38152606	0.37979016	0.38059561	0.38111862
5 R 5/6	0.36490437	0.36458057	0.36418035	0.36482338	0.36365991	0.36392066	0.36423324
7.5 R 5/6	0.35683766	0.35716822	0.35625744	0.35609785	0.35564361	0.35543045	0.35535235
10 R 5/6	0.35625463	0.35672084	0.35627297	0.35609185	0.35620746	0.35609572	0.35607102
2.5 YR 5/6	0.34762214	0.34727648	0.34748020	0.34799130	0.34771222	0.34777714	0.34781263
5 YR 5/6	0.32590123	0.32514523	0.32525658	0.32585322	0.32520233	0.32509455	0.32494030
7.5 YR 5/6	0.30927139	0.30925953	0.30907283	0.30910551	0.30903625	0.30906970	0.30909370
10 YR 5/6	0.27032903	0.27021335	0.26839290	0.26871158	0.26803627	0.26836243	0.26859637
2.5 Y 5/6	0.25562884	0.25547050	0.25483609	0.25405155	0.25384059	0.25387784	0.25428818
5 Y 5/6	0.24245380	0.24226881	0.24263254	0.24098847	0.24116591	0.24096609	0.24149922
7.5 Y 5/6	0.23275924	0.23273598	0.23280456	0.23258277	0.23261385	0.23258369	0.23265184
10 Y 5/6	0.18990586	0.19179698	0.19277121	0.19582143	0.19811129	0.20196074	0.20450251
2.5 GY 5/6	0.16313182	0.16552195	0.16809222	0.17242654	0.17675108	0.18175573	0.18566670
5 GY 5/6	0.13624660	0.13748551	0.13960130	0.14281025	0.14615867	0.14944549	0.15371489
7.5 GY 5/6	0.11360490	0.11503126	0.11685385	0.11968168	0.12212501	0.12523414	0.12838376
10 GY 5/6	0.09973380	0.10119862	0.10279240	0.10531001	0.10717575	0.11004292	0.11246786
2.5 G 5/6	0.08413297	0.08588701	0.08792501	0.09131934	0.09379533	0.09654708	0.09960412
5 G 5/6	0.08003922	0.08134237	0.08328577	0.08606431	0.08852359	0.09162371	0.09522388
7.5 G 5/6	0.08960782	0.09167153	0.09612759	0.10148558	0.10613665	0.11175500	0.11680648
10 G 5/6	0.09024748	0.09318675	0.09664418	0.10290737	0.10904340	0.11482310	0.11963412
2.5 BG 5/6	0.08884098	0.09220722	0.09618193	0.10149670	0.10787549	0.11273278	0.11796647
5 BG 5/6	0.08615588	0.08970830	0.09474591	0.10138868	0.10792587	0.11469073	0.12083654
7.5 BG 5/6	0.08761606	0.09269932	0.09807603	0.10756190	0.11756171	0.12549242	0.13381812
10 BG 5/6	0.08876292	0.09576874	0.10480710	0.11702209	0.13053964	0.14465118	0.15490025
2.5 B 5/6	0.09144159	0.09781734	0.10582335	0.11775719	0.13251715	0.14869438	0.16337824
5 B 5/6	0.10599998	0.11291502	0.12300824	0.13723524	0.15647038	0.18012335	0.20369059
7.5 B 5/6	0.11474568	0.12099406	0.13001795	0.14471691	0.16460293	0.18978842	0.21716613
10 B 5/6	0.12871259	0.13458024	0.14400168	0.15737042	0.17395220	0.19401119	0.21499012
2.5 PB 5/6	0.15689296	0.16338935	0.17124602	0.18030524	0.19145318	0.20456466	0.21711791
5 PB 5/6	0.18340855	0.18911604	0.19669468	0.20667657	0.21616868	0.22883216	0.23856010
7.5 PB 5/6	0.23390607	0.23986951	0.24744079	0.25768084	0.26836382	0.27890805	0.28856743
10 PB 5/6	0.35004715	0.35782847	0.35833517	0.35898220	0.35965816	0.36030589	0.36092067
2.5 P 5/6	0.36611508	0.36666608	0.36667444	0.36668281	0.36669118	0.36669954	0.36670791
5 P 5/6	0.36644057	0.36638042	0.36623424	0.36616274	0.36624711	0.36596801	0.36597529
7.5 P 5/6	0.34086216	0.34056247	0.33881220	0.33823636	0.33899278	0.33629865	0.33662699
10 P 5/6	0.32548147	0.32673443	0.32517524	0.32551180	0.32514992	0.32518790	0.32621568
2.5 RP 5/6	0.36877803	0.36856791	0.36864751	0.36761264	0.36705031	0.36737182	0.36646561
5 RP 5/6	0.39815802	0.39817321	0.39818839	0.39820358	0.39821877	0.39823395	0.39824914
7.5 RP 5/6	0.40474585	0.40444895	0.40376372	0.40345939	0.40386288	0.40345890	0.40468995
10 RP 5/6	0.39972286	0.40018698	0.40007628	0.39939385	0.39843306	0.39888179	0.39998894
3.75 Y 8/14	0.77411066	0.77346479	0.77344048	0.76855240	0.76867538	0.76826275	0.77006123

Table A.1 — Wpt Normalization Spectral Reflectance Factors (continued)

	765nm	770nm	775nm	780nm
10N	1.00000000	1.00000000	1.00000000	1.00000000
2.5 R 5/6	0.38065358	0.37927966	0.37884875	0.37997147
5 R 5/6	0.36437392	0.36276634	0.36236861	0.36324911
7.5 R 5/6	0.35624760	0.35383052	0.35462083	0.35547434
10 R 5/6	0.35682896	0.35559311	0.35642668	0.35694081
2.5 YR 5/6	0.34813566	0.34777634	0.34773936	0.34824817
5 YR 5/6	0.32524003	0.32446245	0.32419377	0.32478681
7.5 YR 5/6	0.30905953	0.30903765	0.30897163	0.30886420
10 YR 5/6	0.26826327	0.26804991	0.26740614	0.26635868
2.5 Y 5/6	0.25473211	0.25416977	0.25352696	0.25380439
5 Y 5/6	0.24255483	0.24173581	0.24112640	0.24247702
7.5 Y 5/6	0.23279570	0.23268935	0.23261564	0.23280671
10 Y 5/6	0.20689218	0.21047153	0.21238654	0.21662566
2.5 GY 5/6	0.18957463	0.19336587	0.19904347	0.20494558
5 GY 5/6	0.15773464	0.16071177	0.16563443	0.17196628
7.5 GY 5/6	0.13191084	0.13548343	0.13976517	0.14609559
10 GY 5/6	0.11556632	0.11925174	0.12300019	0.12901469
2.5 G 5/6	0.10256313	0.10452706	0.10835059	0.11266778
5 G 5/6	0.09735892	0.10097504	0.10564072	0.11047653
7.5 G 5/6	0.11946680	0.12361524	0.12402658	0.12675221
10 G 5/6	0.12448538	0.12592349	0.12809878	0.13117730
2.5 BG 5/6	0.12243453	0.12342095	0.12625933	0.12723474
5 BG 5/6	0.12608089	0.12777318	0.13045129	0.13317117
7.5 BG 5/6	0.13963707	0.14228824	0.14399833	0.14536902
10 BG 5/6	0.16389042	0.16957357	0.17128093	0.17325004
2.5 B 5/6	0.17594571	0.18317969	0.18790146	0.19181394
5 B 5/6	0.22519820	0.24044677	0.25126933	0.25810060
7.5 B 5/6	0.24183717	0.26246743	0.27731408	0.28679958
10 B 5/6	0.23468936	0.25075343	0.26452740	0.27728021
2.5 PB 5/6	0.22710572	0.23618908	0.24726966	0.25805290
5 PB 5/6	0.24746211	0.25715730	0.26616874	0.27439644
7.5 PB 5/6	0.29742353	0.30484634	0.31106442	0.32014468
10 PB 5/6	0.36149388	0.36197848	0.36239719	0.36298774
2.5 P 5/6	0.36671628	0.36672464	0.36673301	0.36674138
5 P 5/6	0.36594872	0.36578490	0.36569026	0.36557542
7.5 P 5/6	0.33587537	0.33400092	0.33339802	0.33205617
10 P 5/6	0.32408405	0.32242839	0.32389632	0.32281635
2.5 RP 5/6	0.36731644	0.36567434	0.36591489	0.36593287
5 RP 5/6	0.39826433	0.39827951	0.39829470	0.39830989
7.5 RP 5/6	0.40399544	0.40168669	0.40356621	0.40346814
10 RP 5/6	0.39908651	0.39607768	0.39807803	0.39898199
3.75 Y 8/14	0.77311789	0.77044810	0.76815450	0.77166775

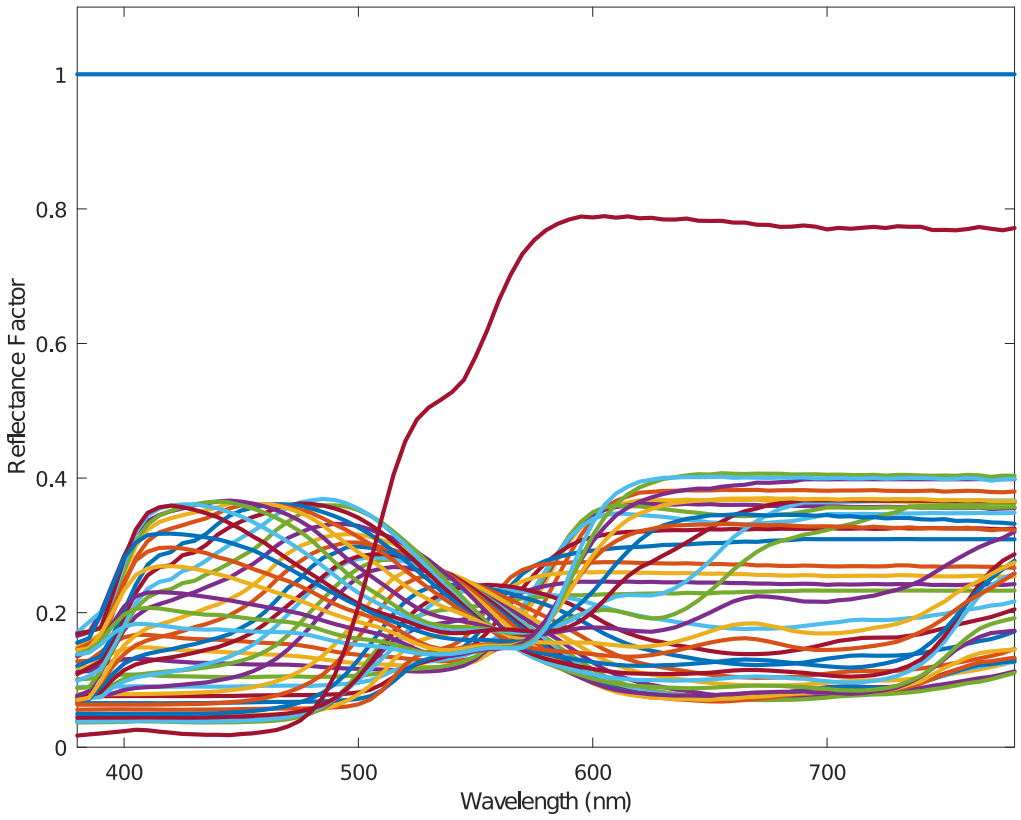


Figure A.1 — Spectral Reflectances from Table A.1

Annex B: Target Wpt Values for Reference Conditions

The following table provides Target Wpt values associated with reflectances in Annex A for the 1931 standard 2-degree observer under CIE Illuminant C to be used as part of Wpt normalization for alternate observing conditions.

Table B.1 — Target Wpt Values for reflectances in Table A.1

	Wpt_W	Wpt_p	Wpt_t
10N	100.000000	0.000000	0.000000
2.5 R 5/6	19.770000	26.695042	10.982492
5 R 5/6	19.770000	26.067546	13.977808
7.5 R 5/6	19.770000	24.960219	16.602502
10 R 5/6	19.770000	22.989691	19.685208
2.5 YR 5/6	19.770000	20.133742	21.813677
5 YR 5/6	19.770000	16.424270	23.883953
7.5 YR 5/6	19.770000	12.796556	25.245781
10 YR 5/6	19.770000	7.993290	26.379308
2.5 Y 5/6	19.770000	3.020043	27.005445
5 Y 5/6	19.770000	-1.781371	27.172161
7.5 Y 5/6	19.770000	-6.274065	26.933974
10 Y 5/6	19.770000	-10.193329	26.208100
2.5 GY 5/6	19.770000	-14.547521	24.622730
5 GY 5/6	19.770000	-18.386090	22.450492
7.5 GY 5/6	19.770000	-22.626282	18.610750
10 GY 5/6	19.770000	-24.945370	14.673577
2.5 G 5/6	19.770000	-26.726738	9.354581
5 G 5/6	19.770000	-27.140988	5.527092
7.5 G 5/6	19.770000	-26.992728	2.684747
10 G 5/6	19.770000	-26.474144	-0.236589
2.5 BG 5/6	19.770000	-25.736723	-3.258910
5 BG 5/6	19.770000	-24.424703	-7.629372
7.5 BG 5/6	19.770000	-23.206719	-11.447218
10 BG 5/6	19.770000	-20.861687	-16.164495
2.5 B 5/6	19.770000	-18.286203	-19.772485
5 B 5/6	19.770000	-14.661893	-23.072061
7.5 B 5/6	19.770000	-10.749886	-25.202760
10 B 5/6	19.770000	-6.763591	-26.378621
2.5 PB 5/6	19.770000	-2.154298	-27.229717
5 PB 5/6	19.770000	2.461019	-27.136567
7.5 PB 5/6	19.770000	8.250615	-26.213663
10 PB 5/6	19.770000	13.249336	-24.246123
2.5 P 5/6	19.770000	16.444117	-21.152198
5 P 5/6	19.770000	19.277634	-17.713687
7.5 P 5/6	19.770000	22.158877	-13.139272
10 P 5/6	19.770000	24.180121	-8.807294
2.5 RP 5/6	19.770000	25.615437	-4.383504
5 RP 5/6	19.770000	26.609042	0.993369
7.5 RP 5/6	19.770000	27.026432	4.819941
10 RP 5/6	19.770000	26.962698	8.161503
3.75 Y 8/14	59.100000	0.000000	100.000000

Annex C: Wpt normalization functions

Target Wpt Values for Reference Conditions

The following MATLAB/GNU Octave function represents an instance of the function f() in eq. 5 for determining Wpt normalization matrices for a reference observing conditions. This function should be called with color matching functions for the 1931 standard 2-degree observer and CIE Illuminant C (which are the observing conditions for the Munsell Renotation data. This function is called to get a normalization matrix to determine Wpt values found in Table B.1 for the reflectances in Table A.1. The function in section C.2 should be utilized for alternate observing conditions.

```
function [ wpts ] = getWptStructReference(obs, illum)
%getWptStruct Gets Wpt (Waypoint) normalization structure
%  wpts = getWptStruct(obs, illum) loads and initializes a structure with
%         Wpt (Waypoint) normalization data
%
%Arguments
% obs = array of column vectors containing sensor sensitivity functions.
%      Wavelengths should range from 380nm to 780nm in 5nm steps.
%      Assumes that sensor excitation values are found using ref2XYZ()
%      which normalizes all values such that 2nd sensor excitation value
%      for PRD is 100. (should be for 1931 standard 2-degree observer)
% illum = illuminant spectral power distribution (SPD) represented as column
%         vector.
%         Wavelengths should match those for the observer.
%
%
%Return value
% wpts = The structure members are as follows:
%
%  lambda:   vector containing wavelengths for all data in structure
%  illum:    vector containing SPD of Wpt normalization illuminant
%  cmf:      array of column vectors containing Wpt normalization sensor
%            sensitivity functions
%  xyz2wpt:  matrix that converts from sensor excitations to Wpt
%            coordinates (as column vectors)
%            Note: This is the same as A in eq. 19
%  wpt2xyz:  matrix that converts from Wpt coordinates to sensor
%            excitations (only defined when there are 3 sensors)
%            Note: This is the same as A^-1 in eq. 19
%  ref2xyz:  matrix that directly direct conversion from reflectance to Wpt
%            coordinates. Assumes that sensor excitation values are found
%            using ref2XYZ() which normalizes all values such that 2nd
%            sensor excitation value for PRD is 100.
%  whiteref: PRD matching lambda wavelengths
%
% Written by Max Derhak - Last updated Dec 2019

lambda = 380:5:780;
if (length(illum)~=length(lambda) || length(obs(:,1))~=length(lambda))
```

```
  error('invalid input data');
end

    %internal function used to convert reflectance to XYZ (sensor excitations)
    function rXYZ = ref2XYZ(_ref, _obs, _illum)
        %first calculate normalization constant
        k = 100 / sum(_obs(:,2).*_illum);

        %calculate XYZ tristimulus vector
        rXYZ = k .* _obs' * diag(_illum) * _ref;
    end


%Load reflectances for Table A.1 as column reflectance vectors
%10 N index 1, hue chips index 2-41, 3.75 Y 8/14 index 42
mR = load('TableA1.mat');

%Use chips with Munsell Value 5 and Munsell Chroma 6
s=2:41;
mCircle = mR(:,s);
opts = optimset('LargeScale', 'off', 'Display', 'off');
cwhite= ref2XYZ(mR(:,1), obs, illum);
cyellow= ref2XYZ(mR(:,42), obs, illum);

%Step 1 - Rotate white point to first (W) axis
lw = cwhite;
phi=-atan2(lw(3), lw(2));
r1 = [1 0 0 ; 0 cos(phi) -sin(phi); 0 sin(phi) cos(phi)];
l1 = r1*lw;
theta = -atan2(l1(2), l1(1));
r3 = [cos(theta) -sin(theta) 0; sin(theta) cos(theta) 0; 0 0 1];
am = (r3 * r1);

%Step 2 - Normalize white to 100 on gray (W) axis
wwpt = am * ref2XYZ(mR(:,1), obs, illum);
am = diag([100/wwpt(1) 1 1]) * am;

%Step 3 - Skew constant Munsell Value plane to be orthogonal to gray (W) axis
Wxy = (am * ref2XYZ(mCircle, obs, illum))';
abc=pinv([Wxy(:,2:3) ones(length(Wxy(:,1)), 1)]) * Wxy(:,1);
am = [1 -abc(1) -abc(2); 0 1 0; 0 0 1] * am;

%Step 4 - rotate & stretch elliptical chroma to be more circular
fm = eye(3);
mr=[];
t=[];
n1=ones(1,length(s));

    %Function used for minimizing fitting to equation of ellipse
    function d=maxellipsedif(v, n1, t, mr)
        a=v(1)*n1;
        b=v(2)*n1;
        rot=v(3);

        ct = cos(t-rot*n1);
        st = sin(t-rot*n1);
```

```
        cr = cos(v(3))*n1;
        sr = sin(v(3))*n1;

        x=a.*ct.*cr-b.*st.*sr;
        y=a.*ct.*sr+b.*st.*cr;

        d=sum(sqrt((x-mr(2,:)).^2 + (y-mr(3,:)).^2));
    end

d=10000;
ld=d+1;
while (ld-d>0.05)
    ld=d;
    mr = fm*am*ref2XYZ(mCircle, obs, illum);
    t = atan2(mr(3,:), mr(2,:));
    ac = mean(sqrt(mr(2,:).^2 + mr(3,:).^2));
    [v d]=fminunc(@(v)(maxellipsedif(v, n1, t, mr)), [ac ac 0], opts);
    rot= -v(3);
    av=mean(v(1:2));
    fm = [1 0 0
          0 av/v(1) 0
          0 0 av/v(2)] * [1 0 0
                         0 cos(rot) -sin(rot)
                         0 sin(rot) cos(rot)] * fm;
end
fm = fm * am;
yel = fm * ref2XYZ(mR(:,42), obs, illum);

%Step 5 - rotate so 3.75Y lies on positive (t) axis
yrot = -atan2(yel(3), yel(2)) + pi()/2;
fm = [1 0 0; 0 cos(yrot) -sin(yrot); 0 sin(yrot) cos(yrot)] * fm;

%Step 6 - normalize chroma scale so 3.75Y is 100
yelN = 100;
yelC = sqrt(sum(yel(2:3).^2));
fm = [1 0 0; 0 yelN/yelC 0; 0 0 yelN/yelC] * fm;

%Step 7 if needed  - flip so red has positive (p)
redwpt = fm * ref2XYZ(mR(:,2), obs, illum);
if (redwpt(2)<0)
    fm = [1 0 0; 0 -1 0; 0 0 1] * fm;
end

%Store results in returned wpts structure
wpts.lambda = lambda;
wpts.illum = illum;
wpts.cmf = obs;
wpts.xyz2wpt = fm; %Represents matrix A_i,k in eq. 5, 6 & 18
wpts.wpt2xyz = inv(wpts.xyz2wpt); %Represents matrix A^-1_i,k in eq. 6 & 18
wpts.ref2wpt = fm * obs' * (diag(illum).*100./(sum(obs(:,2).*illum)));
wpts.whiteref = ones(length(illum), 1);

end
```

Wpt normalization function for alternate conditions

The following MATLAB/Octave function represents an instance of the function f() in eq. 5 the determines Wpt normalization matrices for alternate observers and illuminants.

```
function [ wpts ] = getWptStruct(obs, illum)
%getWptStruct Gets Wpt (Waypoint) normalization structure
%  wpts = getWptStruct(obs, illum) loads and initializes a structure with
%         Wpt (Waypoint) normalization data
%
%Arguments
% obs = array of column vectors containing sensor sensitivity functions.
%       Wavelengths should range from 380nm to 780nm in 5nm steps.
%       Assumes that sensor excitation values are found using ref2XYZ()
%       which normalizes all values such that 2nd sensor excitation
%       value for PRD is 100. This works when obs has 3 or more sensors.
% illum = illuminant spectral power distribution (SPD) represented as column
%         vector.  Wavelengths should match those for the observer..
%
%Return value
% wpts = The structure members are as follows:
%
%  lambda:   vector containing wavelengths for all data in structure
%  illum:    vector containing SPD of Wpt normalization illuminant
%  cmf:      array of column vectors containing Wpt normalization sensor
%            sensitivity functions
%  xyz2wpt:  matrix that converts from sensor excitations to Wpt
%            coordinates (as column vectors)
%            Note: This is the same as A in eq. 19
%  wpt2xyz:  matrix that converts from Wpt coordinates to sensor
%            excitations (only defined when there are 3 sensors)
%            Note: This is the same as A^-1 in eq. 19
%  ref2xyz:  matrix that directly direct conversion from reflectance to Wpt
%            coordinates. Assumes that sensor excitation values are found
%            using ref2XYZ() which normalizes all values such that 2nd
%            sensor excitation value for PRD is 100.
%  whiteref: PRD matching lambda wavelengths
%
% Written by Max Derhak - Last updated Dec 2019

lambda = 380:5:780;
if (length(illum)~=length(lambda) || length(obs(:,1))~=length(lambda))
  error('invalid input data');
end

    %internal function used to convert reflectance to XYZ (sensor excitations)
    function rXYZ = ref2XYZ(_ref, _obs, _illum)
        %first calculate normalization constant
        k = 100 / sum(_obs(:,2).*_illum);

        %calculate XYZ tristimulus vector
        rXYZ = k .* _obs' * diag(_illum) * _ref;
    end

%Load reflectances for Table A.1 as column reflectance vectors
```

```
%10 N index 1, hue chips index 2-41, 3.75 Y 8/14 index 42
mR = load('TableA1.mat');

%Use chips with Munsell Value 5 and Munsell Chroma 6
s=2:41;
mCircle = mR(:,s);
ref = mCircle;
xyz = ref2XYZ(ref, obs, illum);
wxyz = ref2XYZ(mR(:,1), obs, illum);
ndim = length(obs(1,:));
mcone = eye(ndim);

%Load target Wpt values for Table B.1 as column Wpt vectors
tWpt=load('TableB1.mat');
%setup target pt values for regression
mpt = tWpt(2:3,s);
cwhite= mcone * wxyz;

%Step 1 - Rotate white point to first (W) axis
lw = cwhite;
am = eye(ndim);

for d=ndim:-1:2
   phi=-atan2(lw(d), lw(d-1));
   r=eye(ndim);
   r(d-1,d-1) = cos(phi);
   r(d-1, d) = -sin(phi);
   r(d, d-1) = sin(phi);
   r(d, d) = cos(phi);
   lw = r*lw;
   am = r*am;
end

%Step 2 - Normalize white to 100 on (W) gray axis
wWpt = am * mcone * wxyz;
am(1,:) = am(1,:) * (100 / wWpt(1));

%Step 3 - Skew constant Munsell value plane to be orthogonal to gray axis
Wxy = (am * mcone * xyz)';

%Find coefficients for plane containing Munsell value plane
abc=pinv([Wxy(:,2:ndim) ones(length(Wxy(:,1)), 1)]) * Wxy(:,1);
m1 = eye(ndim);
for d=2:ndim
   m1(d,d) = 1/cos(atan(abc(d-1)));
end
m2 = eye(ndim);
for d=2:ndim
   m2(1,d) = -abc(d-1);
end
am = m1 * m2 * am;

%Step 4 - Use regression to get chromatic color transform to match p and t
%values
c = am*mcone*xyz;
mc = mpt * pinv(c(2:ndim, :));
```

```
fm = [am(1,:); mc*am(2:end,:)] * mcone;

%Step 5 Store results in returned wpts structure
wpts.lambda = lambda;
wpts.illum = illum;
wpts.cmf = obs;
wpts.xyz2wpt = fm; % Represents matrix A_i,k in eq. 5, 6 & 18
if ndim==3
   wpts.wpt2xyz = inv(wpts.xyz2wpt); %Represents matrix A^-1_i,k in eq. 6 & 18
end
wpts.ref2wpt = fm * obs' * (diag(illum).*100./(sum(obs(:,2).*illum)));
wpts.whiteref = ones(length(illum), 1);

end
```

Annex D: Wpt to WLab conversion

Making Wpt more perceptually uniform

The following MATLAB/Octave function represents an instance of the function $g()$ in eq. 7 for converting Wpt coordinates to WLab coordinates which are optimized to be more perceptually uniform under D65 for the 1931 standard 2-degree observer.

```
function [ WLab ] = Wpt2WLab( wpt, LargeScaleFlag)
%Wpt2WLab converts wpt coordinates to WLab coordinates based on
%hue adjusted deltaE94 chroma optimization
%
% Arguments:
%  -wpt is an array of wpt vectors
%  -LargeScale flag indicates use of CIELAB chroma optimization
%
% Return:
%  -WLab is an array of WLab vectors
%
% Written by Max Derhak - Last updated Dec 2019

if nargin<2
   LargeScaleFlag = false;
end

   %Linearize perceptual lightness (same as CIEL equations)
   function lw = LinearizeW(w)
      %Set up logical indexing arrays to apply cubic root
      Y=w/100;
      vidx = Y>(6/29)^3;
      notvidx = ~vidx;

      %Apply cubic root to normalized Y values using linear segment for darks
      Y(vidx) = Y(vidx).^(1/3);
      Y(notvidx) = ((1/3)*(29/6)^2) * Y(notvidx) + (16/116);

      lw = 116*Y - 16;
   end

   %Function to convert Cartesian to Polar
   function [ _Wch ] = Wpt2Wch( _Wpt )
      h = atan2(_Wpt(3,:), _Wpt(2,:))*180/pi;
      idx = (h<0);
      h(idx) = h(idx) + 360;
      _Wch = [_Wpt(1,:)
            sqrt(_Wpt(2,:).^2 + _Wpt(3,:).^2)
            h];
   end

   %Function to convert Polar to Cartesian
   function [ _Wab ] = Wch2Wab( _Wch )
```

```
        h = _Wch(3,:) * pi / 180;
        _Wab = [_Wch(1,:)
              cos(h).*_Wch(2,:)
              sin(h).*_Wch(2,:)];
    end

%Conversion uses polar notation
wch = Wpt2Wch(wpt);
wch(1,:) = LinearizeW(wch(1,:));
L5 = 51.576165618406904;
a = 0.026742553710937;
gv = 0.458886718750000;

%make chroma consistent with Lightness
tx=(wch(1,:)-L5);
s=exp(a*tx)/gv;
wch(2,:) = wch(2,:)./s;

%adjust chroma based on hue to match DeltaE94 chroma scale
h=wch(3,:);
sidx = (h>1 & h<181);
cidx = (h>104 & h<284);
r=zeros(1,length(h));
r(sidx) = sin((h(sidx)-1)*pi/180).^3;
r(cidx) = r(cidx)+0.39*cos((h(cidx)-14)*pi/180).^2;
b1 = 62*(r+0.31);
b2 = 0.36*(r+0.19);
wch(2,:) = b1.*(log(wch(2,:)/80+b2)-log(b2));

%undo mean logarithmic chroma correction (Convert WLab to LargeScaleWLab)
if LargeScaleFlag
    %Fitted to CIELAB C
    wch(2,:) = 83.10881*(exp(wch(2,:)./27.97678 + log(0.33601)) - 0.33601);
end

%convert back to cartesian coordinates
WLab = Wch2Wab(wch);
end
```

Annex E: WLab to Wpt conversion

Getting back to Wpt coordinates from a perceptually uniform space

The following Matlab/Octave function represents an instance of the function $g^{-1}()$ in eq. 8 for converting WLab coordinates which are optimized to be more perceptually uniform under D65 for the 1931 standard 2-degree observer to Wpt coordinates.

```
function [ wpt ] = WLab2Wpt( WLab, LargeScaleFlag)
%WLab2Wpt converts uniform WLab coordinates to Wpt coordinates based on
%hue adjusted deltaE94 chroma optimization
%
% Arguments:
%  -WLab is an array of WLab vectors
%  -LargeScale flag indicates use of CIELAB chroma optimization
%
% Return:
%  -wpt is an array of Wpt vectors
%
% Written by Max Derhak - Last updated Dec 2019

if nargin<2
   LargeScaleFlag = false;
end

   %Function to convert Cartesian to Polar
   function [ _Wch ] = Wab2Wch( _Wab )
      h = atan2(_Wab(3,:), _Wab(2,:))*180/pi;
      idx = (h<0);
      h(idx) = h(idx) + 360;
      _Wch = [_Wab(1,:)
            sqrt(_Wab(2,:).^2 + _Wab(3,:).^2)
            h];
   end

   %Function to convert Polar to Cartesian
   function [ _Wpt ] = Wch2Wpt( _Wch )
      h = _Wch(3,:) * pi / 180;
      _Wpt = [_Wch(1,:)
            cos(h).*_Wch(2,:)
            sin(h).*_Wch(2,:)];
   end

%get polar notation
wch = Wab2Wch(WLab);

%redo mean chroma log correction (Convert LargeScaleWLab to WLab)
if LargeScaleFlag
   %Fitted to CIELAB C
   wch(2,:) = 27.97678*(log(wch(2,:)./83.10881 + 0.33601) - log(0.33601));
end

%undo hue based chroma logarithmic adjustment
```

```
h=wch(3,:);
sidx = (h>1 & h<181);
cidx = (h>104 & h<284);
r=zeros(1,length(h));
r(sidx) = sin((h(sidx)-1)*pi/180).^3;
r(cidx) = r(cidx)+0.39*cos((h(cidx)-14)*pi/180).^2;
d0 = 62*(r+0.31);
d1 = 0.36*(r+0.19);
wch(2,:) = 80*(exp(wch(2,:)./d0 + log(d1)) - d1);

%Undo Lightness-chroma normalization
L5 = 51.576165618406904;
a = 0.026742553710937;
gv = 0.458886718750000;
tx=(wch(1,:)-L5);
s=exp(a*tx)/gv;
wch(2,:) = wch(2,:).*s;

   %Delinearize from perceptual lightness (same as CIEL equations)
   function W = DeLinearizeL(v)
      %Set up logical indexing arrays to apply cubic root
      W = (v + 16) / 116;
      vidx = W>6/29;
      notvidx = ~vidx;
      %calculate linear scalar factor
      three_d2 = 3 * (6/29)^2;
      %Apply cube to linear XYZ values using linear segment for darks
      W(vidx) = W(vidx).^3;
      W(notvidx) = (W(notvidx) - (4/29)) .* three_d2;
      W = 100*W;
   end


%Undo linearization of Lightness
wch(1,:) = DeLinearizeL(wch(1,:));

%convert back to cartesian coordinates
wpt = Wch2Wpt(wch);
end
```

References

[von Kries 1970] von Kries J. Influence of adaptation on the effects produced by luminous stimuli. In: MacAdam D.L. (Eds.) *Sources of Color Vision*. MIT Press, Cambridge, MA, pp. 109-119.

[Susstrunk 2005] Susstrunk S. Computing chromatic adaptation. *PhD dissertation*. University of East Anglia.

[Derhak & Berns 2015-1] DERHAK M, BERNS R. 2015. Introducing Wpt (Waypoint): A color equivalency representation for defining a material adjustment transform. *Color Res Appl.*;40(6):535-549.

[Derhak & Berns 2015-2] DERHAK M, BERNS R. 2015. Introducing WLab—Going from Wpt (Waypoint) to a uniform material color equivalency space. *Color Res Appl.*;40(6):550-563

[[Derhak 2015] DERHAK M. 2015. Spectrally Based Material Color Equivalency: Modelling and Manipulation. *PhD thesis*. Rochester Institute of Technology;

[Chau 1999] CHAU W. 1999. Colour Reproduction for Reflective Images. *PhD thesis*. University of Waterloo